Midterm 2 – 2021.11.25.

1.) $\begin{cases} H = \text{damage of the hobbit} & H \sim N(30, 4^2) \\ E = \text{damage of the elf} & E \sim N(30, 4^2) \\ D = \text{damage of the dwarf} & D \sim N(30, 4^2) \end{cases}$

all of them are totally independent

Dr = health of the dragon $\quad Dr \sim N(100, 4^2)$

2p

$\mathbb{P}(\text{we beat the dragon}) = \mathbb{P}(H+E+D > D_r) = \mathbb{P}(\underbrace{H+D+E-D_r}_{\sim N(-10,\, 4^3)} > 0) =$

$= \mathbb{P}\left(\frac{(H+E+D-D_r)-(-10)}{\sqrt{4^3}} > \frac{10}{8}\right) =$ 3p ← it is now standardized

$= 1 - \underbrace{\Phi\left(\frac{10}{8}\right)}_{\approx 0,8944} = \underline{0,1056}$ 2p

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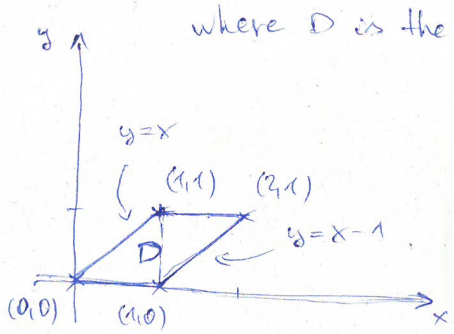
2.) (X, Y) is jointly uniform on D, where D is the paralelogramm



Area of $D = 1$

$\Rightarrow f(x,y) = \begin{cases} 1, & (x,y) \in D \\ 0, & \text{else} \end{cases}$ is the joint density function

$f_X(x) = \int f(x,y)\, dy$ 3p

For $x \in [0,1]$, $f_X(x) = \int_0^x 1\, dy = x$

For $x \in [1,2]$, $f_X(x) = \int_{x-1}^{1} 1\, dy = 2-x$

$\Rightarrow \underline{\underline{f_X(x)}} = \begin{cases} 2-x, & x \in [1,2] \\ x, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$ 3p

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3.) ξ := number of players who gain 2 coins.

$\xi_i := \begin{cases} 1, & \text{if player } i \text{ gain 2 coins} \\ 0, & \text{otherwise} \end{cases}$ 2p $\Rightarrow \xi = \sum_{i=1}^{30} \xi_i$

ξ_i is an indicator r.v.

$\underline{\underline{\mathbb{E}[\xi]}} = \mathbb{E}[\Sigma \xi_i] = \sum_{i=1}^{30} \mathbb{E}[\xi_i] = \sum_{i=1}^{30} \underbrace{\mathbb{P}(\xi_i = 1)}_{=1/9} =$

$= 30 \cdot \frac{1}{9} = \frac{10}{3} \approx \underline{3,3\dot{3}}$ 3p

$\mathbb{P}(\xi_i = 1) = \frac{1}{3} \cdot \frac{1}{9} + \frac{1}{3} \cdot \frac{1}{9} + \frac{1}{3} \cdot \frac{1}{9}$ 2p

we choose rock ↑ both neighbours choose scissors

Bonus/ $\mathrm{Var}(\xi) = \mathbb{E}[\xi^2] - \mathbb{E}^2[\xi]$

$$\mathbb{E}[\xi^2] = \mathbb{E}\left[\left(\sum_{i=1}^{30} \xi_i\right)^2\right] = \sum_{i=1}^{30} \mathbb{E}[\xi_i^2] + 2\sum_{i<j} \mathbb{E}[\xi_i \xi_j]$$ 1p

$$\mathbb{E}[\xi_i^2] \underset{\uparrow}{=} \mathbb{E}[\xi_i] = \frac{1}{9}$$

it's an indicator
$1^2 = 1,\ 0^2 = 0$

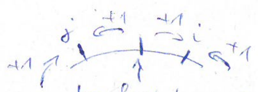
$\mathbb{E}[\xi_i \xi_j] =$

$j = i+1$: $\mathbb{P}(\xi_i \xi_j = 1) = 0$, since one of them must beat the other, who lose a coin then.

$(|j - i| = 2)$

$j = i+2$: $\mathbb{P}(\xi_i \xi_j = 1) = \sum_{k=1}^{3} \frac{1}{3} \frac{1}{3} \cdot \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^4$

2p



he lost to both $\Rightarrow$ i and j chose the same hand

$j > i+2$: $\mathbb{P}(\xi_i \xi_j = 1) \underset{\uparrow}{=} \mathbb{P}(\xi_i = 1)\,\mathbb{P}(\xi_j = 1) = \left(\frac{1}{9}\right)^2 = \left(\frac{1}{3}\right)^4$

independence

$$\Rightarrow \underline{\underline{D^2[\xi^2]}} = \underbrace{30 \cdot \frac{1}{9} + 2\sum_{\substack{i,j=1 \\ j \neq i+1}}^{30} \left(\frac{1}{3}\right)^4}_{= \mathbb{E}[\xi^2]} - \underbrace{\left(\frac{10}{3}\right)^2}_{= \mathbb{E}^2[\xi]} = \frac{10}{3} - \left(\frac{10}{3}\right)^2 + 2\left(\underbrace{\binom{30}{2}}_{= 29 \cdot 15} - 29\right)\left(\frac{1}{3}\right)^4 \approx \underline{\underline{2{,}2469}}$$ 1p