Matehamtics for Civil Engineers, Sample Final Exam, 2014

1. Consider the equation $u_{tt} = u_{xx}$ that describes the vibration of an *infinite* string. It is known that $u(x,0) \equiv 0$ and $u_t(x,0) = \sin x$. $u(x,t) =?$

2. Let $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$. Find a base for the subspace row(A)!

3. Let $B = \begin{bmatrix} 9 & 6 \\ 6 & 9 \end{bmatrix}$. Find a positive definite matrix C such that: $B = C^2$.

4. Test whether the following vector field is a gradient vector field and if yes, then determine the potential! $\overrightarrow{F}(x,y,z) = (1 + 4y + 5z, 2 + 4x, 3 + 5x)$.

5. Let $\overrightarrow{G}(x,y,z) = (2x + y^2, y + \sin z, z - x^3)$, while $\mathcal{F}$ is the surface of the unit sphere about the origin, oriented with an outward pointing normal vector. Use Gauss theorem to evaluate the following surface integral: $\iint_{\mathcal{F}} \overrightarrow{G} d\overrightarrow{A} =?$

6. Let the vector field $\overrightarrow{F}(x,y,z) = (z^2, x + x^2 + z^2, y^2 + x)$ describe the (stationary) velocity field of a fluid. An infinitesimally small paddle wheel is placed with center located at the point $P = (1,2,3)$. Let $\overrightarrow{n}$ denote the normal vector orthogonal to the plane of the paddle wheel. For which choice of $\overrightarrow{n}$ can it be achieved that the paddle wheel rotates with the highest possible speed in the counterclockwise direction when it is observed from the direction $-\overrightarrow{n}$?