Name:.. Neptun code:..................

Advanced mathematics for civil engineers *Group A*

I. exam, 21st December 2021, 9:10-10:40

1. *(15 points)*
   (a) Find the matrix of the orthogonal projection from $\mathbb{R}^2$ to the line $y = -\frac{x}{3}$.
   (b) What is the orthogonal projection of the point $P = (1, 2)$ to this line?

2. *(15 points)* Let
$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$
   Find the matrix e^A.

3. *(15 points)* Let $u(x,t)$ be the function which describes the vibration of the infinite vibrating string, which satisfies the equations $u''_{tt} = 4u''_{xx}$ $(t > 0, x \in \mathbb{R})$,
$$u(x,0) = 0 \text{ for } x \in \mathbb{R}, \text{ and } u'_t(x,0) = \begin{cases} 3 & \text{if } 0 \le x \le 10, \\ 0 & \text{otherwise.} \end{cases}$$
   What is $u(-1,2) =?$

4. *(15 points)* Let $\overrightarrow{F}(x,y,z) = (4x^3y^3z^2 + 2xz, 3x^4y^2z^2, 2x^4y^3z + x^2)$.
   (a) With the Curl-test decide whether $\overrightarrow{F}$ is conservative or not! If it is, determine the potential function!
   (b) Let γ be straight line from point $C = (1,-1,1)$ to $D = (-1,2,2)$. Calculate $\int_\gamma \overrightarrow{F} d\mathbf{r} =?$

5. *(15 points)* Let $\mathcal{F} = \{(x,y,z) : x^2 + y^2 = \frac{1}{9}, 0 \le z \le 1\}$ be the cylinder wall with orientation pointing away from the z-axis. (Note that $\mathcal{F}$ does not contain the disks on the top and on the bottom!) Let $\overrightarrow{G}(x,y,z) = (xz + e^y\cos(z), yz + \sin(x^3 + z^4), 1 - z^2)$. By using Gauss' Theorem, find $\iint_{\mathcal{F}} \overrightarrow{G} d\overrightarrow{A} =?$

6. *(15 points)* Consider the closed curve γ: $r(\varphi) = 2\sin(\varphi)$, $0 \le \varphi \le \pi$ on the plane represented in polar coordinates (i.e. $\mathbf{r}(t) = (2\sin(t)\cos(t), 2\sin^2(t))$, $0 \le t \le \pi$). By using Green's Theorem, find the area of the domain surrounded by γ. (Hint: use the vectorfield $\overrightarrow{F}(x,y) = (0,x)$.)

Name:.. Neptun code:..................

Advanced mathematics for civil engineers *Group B*

I. exam, 21st December 2021, 9:10-10:40

1. *(15 points)*
   (a) Find the matrix of the orthogonal projection from $\mathbb{R}^2$ to the line $y = 3x$.
   (b) What is the orthogonal projection of the point $P = (-2, 1)$ to this line?

2. *(15 points)* Let
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}.$$
   Find the matrix $\cos(A)$.

3. *(15 points)* Let $u(x,t)$ be the function which describes the vibration of the infinite vibrating string, which satisfies the equations $u''_{tt} = 9u''_{xx}$ $(t > 0, x \in \mathbb{R})$,
$$u(x,0) = 0 \text{ for } x \in \mathbb{R}, \text{ and } u'_t(x,0) = \begin{cases} 2 & \text{if } 0 \le x \le 5, \\ 0 & \text{otherwise.} \end{cases}$$
   What is $u(-1,2) =$?

4. *(15 points)* Let $\overrightarrow{F}(x,y,z) = (3x^2y^4z, 4x^3y^3z + z^2, x^3y^4 + 2yz)$.
   (a) With the Curl-test decide whether $\overrightarrow{F}$ is conservative or not! If it is, determine the potential function!
   (b) Let γ be straight line from point $C = (1, -1, 1)$ to $D = (-1, 2, 0)$. Calculate $\int_\gamma \overrightarrow{F} d\mathbf{r} =$?

5. *(15 points)* Let $\mathcal{F} = \{(x,y,z) : x^2 + y^2 = \frac{1}{4}, 0 \le z \le 1\}$ be the cylinder wall with orientation pointing away from the z-axis. (Note that $\mathcal{F}$ does not contain the disks on the top and on the bottom!) Let $\overrightarrow{G}(x,y,z) = (xz + e^y \cos(z), yz + \sin(x^3 + z^4), z^2)$. By using Gauss' Theorem, find $\iint_{\mathcal{F}} \overrightarrow{G} \, d\overrightarrow{A} =$?

6. *(15 points)* Consider the closed curve γ: $r(\varphi) = 2\cos(\varphi)$, $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$ on the plane represented in polar coordinates (i.e. $\mathbf{r}(t) = (2\cos^2(t), 2\cos(t)\sin(t))$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$). By using Green's Theorem, find the area of the domain surrounded by γ. (Hint: use the vectorfield $\overrightarrow{F}(x,y) = (0, x)$.)