Name:.. Neptun code:.................

Advanced mathematics for civil engineers *Group A*

IIIrd exam, 11th January 2022, 9:10-10:40

1. *(15 points)* Find the equation of the line, which fits the best (in the sense of least squares) to the points $(-4,4),(-2,2),(0,1),(1,-1)$ and $(5,-7)$! What is the square error?

2. *(15 points)* Draw the points on the plane, which satisfy the equation $37x^2+18xy+13y^2=40$.

3. *(15 points)* Consider the following heat transport equation!
$$\begin{cases} u'_t = 4u''_{xx} & 0 \le x \le 3, t \ge 0; \\ u(0,t) = u(3,t) = 0 & t \ge 0; \\ u(x,0) = \sin(\pi x) + \cos(\frac{\pi}{3}x)\sin(\frac{2\pi}{3}x) & 0 \le x \le 3. \end{cases}$$
Determine the function $u(x,t)$!

4. *(15 points)* Consider the planar vectorfield $\overrightarrow{F}(x,y) = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2}\right)$.
   (a) With the planar Curl-test decide whether $\overrightarrow{F}$ is conservative or not! If the answer is positive, determine the potential function!
   (b) Let $\gamma : \mathbf{r}(t) = (\cos t, \sin t);\ 0 \le t \le 2\pi$ be the closed unit circle centered at the origin. Calculate $\int_\gamma \overrightarrow{F} d\mathbf{r} = ?$

5. *(15 points)* Denote $\mathcal{F}$ the triangle formed by the points $A(2,-1,5)$, $B(5,-1,5)$ and $C(2,1,5)$ with orientation pointing upwards. Moreover, let $\overrightarrow{F} = (1,4,-3)$ be a constant vectorfield. Find $\iint_{\mathcal{F}} \overrightarrow{F} d\overrightarrow{A} = ?$

6. *(15 points)* Determine the line integral of the planar vectorfield $\overrightarrow{G}(\mathbf{r}) = \overrightarrow{G}(x,y) = (2y, 3x)$ over the closed curve $(x-1)^2+(y-4)^2=16$! (Hint: You might use the definition but with Green's Theorem it has a simpler solution.)