Name:..... Neptun code:.....

Probability Theory 1

I. Midterm test 2018.10.11.

Max point: 40, Duration: 90 min

1. On the table of Albus Dumbledore, there are 30 Bertie Botts Every Flavour Beans. Out of them, 10 have strawberry flavour, 9 chocolate, 6 spinach, 4 liver and 1 ghost flavoured. Harry takes 8 of them.
   a) *(3 points)* What is the probability that all that Harry took have the same flavour?
   b) *(7 points)* Harry took at least one from all the flavours?

2. The 60% of a long book is produced by Press A and 40% by Press B. On average, in Press A they make 2 tyops in every 20 pages and in Press B, 3 typos per 20 pages. I bought one copy of the book.
   a) *(5 points)* What is the probability that I will find at least 4 typos in the first 40 pages?
   b) *(5 points)* I read the first 20 pages and I found exactly 2 typos, what is the probability that the book is from Press A?

3. Amelie and Brigitte play with two dices. If the sum of the results is 7, Brigitte pays to Amelie 10 Euros. If the sum is 6 then Amelie pays x Euros to Brigitte. If the sum is neither 6 nor 7 then they continue the throwing until the resulting sum becomes either 6 or 7.
   (a) *(4 points)* How should they choose x to make the game fair?
   (b) *(6 points)* Amelie has a proposal. Fix $x = 100$ and in every odd step Amelie is the winner with 7 and Brigitte with 6 but in every even step Brigitte wins with 7 and Amelie with 6. Is it a fair game now? Find the expected value of the prise of Amelie!
   (c) *(5 points)* What is the probability that Brigitte wins with this new version?

4. *(5 points)* We throw a fair coin 4 times consecutively. Let the random variable X be the number of clear subsequences. A clear subsequence is a maximal subsequence of throws in which the results are all the same. For example, if the result is $HHTH$ then HH,T and H are the clear subsequences and then $X = 3$, if the result is $HHHH$ then $X = 1$, since all the throws had the same result. Describe the distribution of X! Calculate the expected value and the variance!

Extra: *(5 points)* Describe the distribution of X if the number of throws is n.