Name:..... Neptun code:.....

Probability Theory 1

Sample exam 20??.??.??

Max point: 100, Duration: 100 min

Theo. 1. *(12 points)* Let X be a random variable with uniform distribution on $(0,1)$. For which non-decreasing functions f will the random variable $f(X)$ have uniform distribution on an interval? Verify your answer!

Theo. 2. Let X, Y be random variables and let $y \in \mathbb{R}$ be fixed. Describe
- **a)** *(4 points)* the conditional probabilities in the discrete case;
- **b)** *(4 points)* the conditional distribution function in the jointly continuous case;
- **c)** *(4 points)* the conditional denstity function in the jointly continuous case

under the condition $Y = y$. If there were conditions on events with probability 0 in your answer, explain how to understand it!

Theo. 3.
- **a)** *(10 points)* Define the distribution function F of a random variable X! What are the properties of F? Prove these properties!
- **b)** *(6 points)* Let Y be an absolutely continuous non-negative random variable with finite expected value. Show that $\mathbb{E}(Y) = \int_0^\infty \mathbb{P}(Y > y)dy$!

Prac. 1. Throw ten times a fair dice.
- **a)** *(10 points)* What is the probability that $1, 2, \ldots, 6$ occur at least once?
- **b)** *(10 points)* Denote X the number of different results during the ten throws. (For example, if all the six numbers occur then $X = 6$, if all the throws are 4 then $X = 1$.) What is $\mathbb{E}(X) =$?

Extra: *(10 points)* $\mathbb{D}^2(X) =$?

Prac. 2. Throw a fair coin. If it is a head then choose randomly and uniformly a point in the rectangle with vertices $(0,0); (1,0); (1,2); (0,2)$. If the coin is a tail then choose a point in the rectangle $(0,0); (2,0); (2,1); (0,1)$ uniformly. Denote (X, Y) the coordinates of the random point chosen in this way.
- **a)** *(4+8 points)* Find $\mathbb{E}(X)$ and $\mathbb{D}^2(X)$!
- **b)** *(8 points)* $Cov(X; Y) =$? (Hint: Check the expected value of $X \cdot Y$ by conditioning on the result of the coin tossing.)

Prac. 3. The spaceship of the rebels (the Likelihood Eagle) is under attack by the droid army of the evil Deterministic Empire. The Empire is shooting the Eagle with a lot of laser beams but the Eagle can fend off most of them. However, there are two hits per hour in average.
- **a)** *(6 points)* What is the probability that the Eagle receives three hits in one and half hours?
- **b)** *(14 points)* The fortification arrives to help the Eagle after 55 hours. What is the probability that the Eagle was hit by at most 100 laser beams? Use the Central Limit Theorem to estimate the probability! (On the other side of the sheet, you can find the distribution table of the standard normal variable.)

TABLE 5.1: AREA $\Phi(x)$ UNDER THE STANDARD NORMAL CURVE TO THE LEFT OF X

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998