Name:..... Neptun code:.....

Advanced mathematics for civil engineers *Group A*

Midterm test, 30 October 2017, 10:15-11:00

1. *(15 points)* Let

$$A = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 2 & 1 & 1 & -4 \\ 3 & 2 & 4 & -11 \end{bmatrix}.$$

   a) Determine $\operatorname{rank}(A), \operatorname{rank}(A^T A), \operatorname{nullity}(A)$!
   b) Find a basis of $\operatorname{col}(A)$ formed by the column vectors of A!

2. *(15 points)* Find the equation of the line, which fits the best (in the sense of smallest squares) to the points $(1, 2), (3, -1), (0, 2), (-1, 4)$!

3. *(15 points)* Let

$$B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}.$$

   Find the singular value decomposition of B!

4. *(15 points)* The vibration of a string with length 6 is determined by the equation $u''_{tt} = 4u''_{xx}$. We hit on the left-hand side from below, and on the right-hand side from above with a hammer of length 3 with unit velocity. (That is, $u(0, x) = 0$ and

$$u'_t(0, x) = \begin{cases} 1 & \text{if } x \in [0, 3] \\ -1 & \text{if } x \in (3, 6]. \end{cases})$$

   Describe the vibration of the string!

Name:..... Neptun code:.....

Advanced mathematics for civil engineers *Group B*
Midterm test, 30 October 2017, 10:15-11:00

1. *(15 points)* Let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 0 & 1 & 4 \\ 3 & 2 & 2 \end{bmatrix}.$$

a) Determine $\operatorname{rank}(A), \operatorname{rank}(A^T A), \operatorname{nullity}(A)$!
b) Find a basis of $\operatorname{row}(A)$ formed by the row vectors of A!

2. *(15 points)* Find the equation of the line, which fits the best (in the sense of smallest squares) to the points $(4,1), (-4,0), (0,3), (1,1)$!

3. *(15 points)* Let

$$B = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}.$$

Find a matrix C such that $C^2 = B$!

4. *(15 points)* The vibration of a string with length 6 is determined by the equation $u''_{tt} = u''_{xx}$. We hit on the left-hand side from below, and on the right-hand side from above with a hammer of length 3 with unit velocity. (That is, $u(0,x) = 0$ and

$$u'_t(0,x) = \begin{cases} 1 & \text{if } x \in [0,3] \\ -1 & \text{if } x \in (3,6]. \end{cases})$$

Describe the vibration of the string!

Name:..... Neptun code:.....

Advanced mathematics for civil engineers *Group C*
Midterm test, 30 October 2017, 11:15-12:00

1. *(15 points)* Let

$$A = \begin{bmatrix} 1 & 0 & -2 & 3 & 0 \\ 2 & 1 & 1 & -4 & 0 \\ 3 & 2 & 4 & -11 & 2 \\ 3 & 1 & -1 & -1 & 2 \end{bmatrix}.$$

   a) Determine $\operatorname{rank}(A), \operatorname{rank}(A^T A), \operatorname{nullity}(A)$!
   b) Find a basis of $\operatorname{col}(A)$ formed by the column vectors of A!

2. *(15 points)* Find the equation of the line, which fits the best (in the sense of smallest squares) to the points $(1,0), (2,1), (-2,0), (-3,2)$!

3. *(15 points)* Let

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}.$$

   Find the singular value decomposition of B!

4. *(15 points)* The vibration of a string with length 4 is determined by the equation $u''_{tt} = 4u''_{xx}$. We hit on the left-hand side from below, and on the right-hand side from above with a hammer of length 2 with unit velocity. (That is, $u(0,x) = 0$ and

$$u'_t(0,x) = \begin{cases} 1 & \text{if } x \in [0,2] \\ -1 & \text{if } x \in (2,4]. \end{cases})$$

   Describe the vibration of the string!

Name:..... Neptun code:.....

Advanced mathematics for civil engineers *Group D*
Midterm test, 30 October 2017, 11:15-12:00

1. *(15 points)* Let

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 2 & 1 & 0 & 3 \\ 0 & 1 & 4 & 3 \\ 3 & 2 & 2 & 6 \end{bmatrix}.$$

   a) Determine $\operatorname{rank}(A), \operatorname{rank}(A^T A), \operatorname{nullity}(A)$!
   b) Find a basis of $\operatorname{row}(A)$ formed by the row vectors of A!

2. *(15 points)* Find the equation of the line, which fits the best (in the sense of smallest squares) to the points $(0,2), (0,3), (-2,1), (1,1)$!

3. *(15 points)* Let

$$B = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}.$$

   Find a matrix C such that $C^2 = B$!

4. *(15 points)* The vibration of a string with length 8 is determined by the equation $u''_{tt} = 9u''_{xx}$. We hit on the left-hand side from below, and on the right-hand side from above with a hammer of length 4 with unit velocity. (That is, $u(0,x) = 0$ and

$$u'_t(0,x) = \begin{cases} 1 & \text{if } x \in [0,4] \\ -1 & \text{if } x \in (4,8]. \end{cases})$$

   Describe the vibration of the string!