Thesis summary

In this thesis we introduced and discussed the notion of spindle convexity and ball polyhedra in the Euclidean space $\mathbb{R}^n$. We also discussed separation of spindle convex sets with unit balls. We defined *spherical convexity* and showed how *Carátheodory's theorem* can be reformulated for spherically convex sets. Our main focus of the thesis was the proof of *Steinitz's theorem* for a special class of ball polyhedra, called *standard ball polyhedra*, defined as follows.

Let $X \subset \mathbb{R}^n$ be a non-empty, finite set with circumradius, $\mathrm{cr}(X) \leq 1$, then a ball polyhedron P is defined as the intersection of closed unit balls[1] centered at points of X, that is,

$$P := \bigcap_{x \in X} \mathbf{B}^n[x, 1].$$

The investigation of the properties of ball polyhedra is a rich and complex area of geometry with many unexpected results; in particular, we saw how a ball polyhedron's face-structure is not necessarily a lattice. This motivated the need to restrict our study to standard ball polyhedra, defined as the ball polyhedron whose faces form a lattice with respect to containment. Steinitz's theorem states that a graph $G = (V, E)$ is the edge-graph of a convex polyhedron P, if and only if, the graph G is simple, planar and 3-connected. The version we proved says that a graph G is the edge-graph of a standard ball polyhedron $P \subset \mathbb{R}^3$, if and only if, the edge-graph $G(P)$ is simple, planar and 3-connected.

The combinatorial[2] part of the proof of Steinitz's theorem is only mentioned as a reference and we do not focus on it. We focus on the geometric part. For the proof, we reviewed also some elementary graph theory.

[1] We restrict our definition to unit balls but this definition can be extended to any radius $r > 0$.

[2] The "if" part of the proof.