Budapest University of Technology and Economics
Institute of Mathematics
Department of Stochastics

Abstract

Limit shape of non-intersecting Brownian bridges

Zsolt Bartis

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The aim of this thesis is to investigate the limit-shape of non-intersecting Brownian motions in the plane and their fluctuations. This topic has attracted considerable attention in recent years due to its relevance in various fields such as statistical mechanics, combinatorics, and probability theory.

The study of non-intersecting Brownian motions has a rich history, dating back to the pioneering work of F. Dyson in 1962, which is a seminal contribution to the study of random matrices and the use of Brownian motion models in physics and mathematics. This breakthrough result has led to a deeper understanding of the properties of non-intersecting Brownian motions which can be represented as the eigenvalue process of a random matrix.

In this thesis we provide an insight to the various mathematical tools needed for our research, such as probability theory, matrix algebra and differential equations. Then, we go further and analyze the behaviour of the particles from a non-intersecting Brownian bridge setting. We were particularly curious about the shape of that region, where these particles are located in space-time, and we are able to describe what kind of set they fill with the help of the complex Burger's equations that we solve for a particular initial and final condition.

Finally, we dive deeper into the numerical approach of the mathematical concepts presented earlier. The use of computer simulations allows us to see the results graphically and gain a better understanding of the underlying theory.