Practice Problems $\sharp 1$

1. Find the angle between $\mathbf{a}$ and $\mathbf{b}$, the scalar component of $\mathbf{b}$ in the direction of $\mathbf{a}$, the vector projection of $\mathbf{b}$ onto $\mathbf{a}$, and then write $\mathbf{b}$ as a sum of a vector parallel and a vector orthogonal to $\mathbf{a}$.

   (a) $\mathbf{a} = (0, 5, -3)$, $\mathbf{b} = (1, 1, 1)$

   (b) $\mathbf{a} = (2, -4, \sqrt{5})$, $\mathbf{b} = (-2, 4, -\sqrt{5})$

2. Given the points $A(2, 1, 0)$, $B(2, -1, 1)$, and $C(1, 0, 2)$, find the (a) area of the triangle ABC; (b) the volume of the tetrahedron $OABC$; (c) the coordinates of vertex D in the parallelogram $ABCD$.

3. Find the equation of the plane through $(2, 4, 5)$, $(1, 5, 7)$, and $(-1, 6, 8)$. Find the distance of the point $P(1, 2, 3)$ from this plane. Find the equation of the line through P orthogonal to the plane. Find the orthogonal projection P' of P to the plane.

4. Find the equation for the line in which the planes $x - 2y + 4z = 2$ and $x + y - 2z = 5$ intersect.

5. Find the equation of the line which is the orthogonal projection of the line $x = 1 - t$, $y = 3t$, $z = 1 + t$ to the plane $2x - y + 3z = 6$.

6. Is the following implication true? If $\mathbf{a} \cdot \mathbf{b_1} = \mathbf{a} \cdot \mathbf{b_2}$ and $\mathbf{a} \neq \mathbf{0}$, then $\mathbf{b_1} = \mathbf{b_2}$. If yes, prove it, if not, give an example.

7. Is the following implication true? If $\mathbf{a} \times \mathbf{b_1} = \mathbf{a} \times \mathbf{b_2}$ and $\mathbf{a} \neq \mathbf{0}$, then $\mathbf{b_1} = \mathbf{b_2}$. If yes, prove it, if not, give an example.