Homework Exercises $\sharp 13$

1. Find the global maximum and minimum of the function $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate

$$D = \{(x, y) : 0 \leq x \leq 1, \quad 2x \leq y \leq 2\}.$$

2. A flat circular plate has the shape of the region

$$D = \{(x, y) : x^2 + y^2 \leq 1\}.$$

The plate is heated so that the temperature at the point (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Sketch an assortment of level curves of T (so-called isotherms) in D. Find the temperature at the hottest and coldest points on the plate.

3. The potential of a field is $u(x, y) = xy + 2x - \ln x^2 y$ in the open first quadrant

$$D = \{(x, y) : x > 0, y > 0\}.$$

Show that the potential has a single minimum point and no maximum point in D, tends to $+\infty$ as $x \to 0+$ or $y \to 0+$ or $x \to +\infty$ or $y \to +\infty$.

4. Find the global maximum and minimum values of $f(x, y) = xy$ on the curve $x^2 + 4y^2 = 8$. Sketch the curve and also some level curves of the function.

5. A space probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe's surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface. (Hint: You may use the method of Lagrange multiplier for a three-variable function.)

6. Evaluate the integral

$$\iint_R (x^2 y - 2xy)\, dx\, dy$$

over the rectangle $R = [-2, 0] \times [0, 3]$.

7. Evaluate the integral

$$\iint\limits_R y \cos xy \, dx \, dy$$

over the rectangle $R = [0, \pi] \times [0, 1]$.

8. Evaluate the integral

$$\iint\limits_D (x^2 y - 2xy) \, dx \, dy$$

over the domain

$$D = \{(x, y) : 1 \leq x \leq 2, \quad x \leq y \leq 2x\}.$$

9. Find volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top is bounded by the plane $z = x + 4$.

10. Evaluate the integral by reversing the order of integration:

$$\int\limits_0^\pi \int\limits_x^\pi \frac{\sin y}{y} \, dy \, dx.$$