# Homework Exercises ♯1

1. Determine whether the improper integrals converge or diverge. (You do not have to evaluate the convergent ones.)

$$\text{(a)} \int_0^\infty \frac{dx}{1+e^x} \quad \text{(b)} \int_{-1}^1 \frac{dx}{x^2} \quad \text{(c)} \int_0^\infty \frac{dx}{\sqrt{x}} \quad \text{(d)} \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$\text{(e)} \int_0^1 \frac{dx}{\sqrt{x-x^2}} \quad \text{(f)} \int_0^\infty \frac{dx}{\sqrt{x+x^4}} \quad \text{(g)} \int_0^\infty \frac{dx}{\sqrt{x^6+1}} \quad \text{(h)} \int_0^\infty x^2 e^{-x}\, dx$$

$$\text{(i)} \int_1^\infty \frac{\sqrt{x+1}}{x^2}\, dx \quad \text{(j)} \int_\pi^\infty \frac{2+\cos x}{x}\, dx \quad \text{(k)} \int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$$

2. Estimate the value of the improper integral $\int_0^\infty e^{-x^2}\, dx$. (Hint: Use Simpson's rule with $n = 6$ for $\int_0^3 e^{-x^2}\, dx$ and show that

$$0 < \int_3^\infty e^{-x^2}\, dx < \int_3^\infty e^{-3x}\, dx < 10^{-4}.)$$

3. Find the values of $p$ for which each integral converges:

$$\text{(a)} \int_1^2 \frac{dx}{x(\ln x)^p} \qquad \text{(b)} \int_2^\infty \frac{dx}{x(\ln x)^p}.$$

4. *(Gabriel's horn)*
Revolve the curve $y = 1/x$, $1 \le x < \infty$ about the $x$-axis. Show that the resulting solid has finite volume but infinite surface area. ("This is a can that does not hold enough paint to cover its outside surface".)

5. Determine whether the numerical series converge or diverge. Evaluate the convergent ones.

$$\text{(a)} \sum_{n=0}^\infty \left(\frac{1}{\sqrt{2}}\right)^n \quad \text{(b)} \sum_{n=0}^\infty \cos n\pi \quad \text{(c)} \sum_{n=1}^\infty (-1)^n n \quad \text{(d)} \sum_{n=1}^\infty \frac{2^n-1}{3^n}$$

$$\text{(e)} \sum_{n=1}^\infty \left(1-\frac{1}{n}\right)^n \quad \text{(f)} \sum_{n=1}^\infty (-1)^n x^n, \quad |x|<1; \qquad \text{(g)} \sum_{n=1}^\infty (-1)^n x^{2n}, \quad |x|<1.$$

6. Use partial fractions to find a formula for the $n$th partial sum and use it to find the sum of the series.

$$\text{(a)} \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}.$$

7. Show that if $\sum a_n$ converges, and $a_n \neq 0$ for all $n$, then $\sum(1/a_n)$ diverges.