Homework Exercises ♯8

1. Use the definition to find the derivative of the function $f(x) = \sqrt{x}$ for $x > 0$. Write an equation of the tangent at $x = 3$.

2. Is the function differentiable at $x = 0$? (Hint: Use right-hand and left-hand derivatives.)

$$f(x) = \begin{cases} x^2 & \text{for } x < 0, \\ x & \text{for } x \geq 0. \end{cases}$$

3. Compute the first and second derivatives of the function $f(x) = |x|^3$ for all real x, and show that $f'''(0)$ does not exist.

4. Find the first and second derivatives of the functions.

   (a) $f(x) = \frac{x^2}{1-x^3}$

   (b) $f(x) = \frac{(x-1)(x^2+x+1)}{x^3}$ (Hint: First simplify.)

   (c) $f(x) = \frac{\cos x}{1+\sin x}$

5. Suppose u and v are functions of x that are differentiable at $x = 0$ and that

$$u(0) = 5 \quad u'(0) = -3 \quad v(0) = -1 \quad v'(0) = 2.$$

   Find the values of the following derivatives at $x = 0$:

$$(uv)', \quad \left(\frac{u}{v}\right)', \quad \left(\frac{v}{u}\right)', \quad (7v - 2u)'.$$

6. Find the tangents to the curve $y = x^3 + x$ at the points where the slope is 4. What is the smallest slope on the curve? At what value of x does the curve have this slope?

7. Do the graphs of the functions have any horizontal tangents in the interval $0 \leq x \leq 2\pi$? If so, where? If not, why not?

   (a) $y = x + \sin x$ (b) $y = 2x + \sin x$

8. If the gas in a cylinder is maintained at a constant temperature T, the pressure p is related to the volume V by a formula of the form

$$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2},$$

   in which a, b, n, and R are constants. Find dp/dV.

9. (Marginal revenue) Suppose the revenue from selling x custom-made office desk a week is

$$r(x) = 2000\left(1 - \frac{1}{x+1}\right)$$

dollars.

   (a) Find the marginal revenue when x desks are produced a week.

   (b) Use the function $r'(x)$ to estimate the increase in revenue that will result from increasing production from 5 desks a week to 6 desks a week.

   (c) Find the limit of $r'(x)$ as $x \to \infty$. How would you interpret this number?

10. Solve the the following graphical problems form Calculus 3.3: 19 and 27.