Homework Exercises ♯14

1. Find the area enclosed by the curves $y = \cos(\pi x/2)$ and $y = 1 - x^2$. First sketch the graphs.

2. Find the volume of the solid generated by revolving about the x-axis the region enclosed by the lines $x = 0$, $y = 2$, and the curve $y = e^{x/2}$. First sketch the graphs.

3. Find the length of the "star-shaped" curve called astroid given by the equation $x^{2/3} + y^{2/3} = 1$. First sketch the graph.

4. Find the area of the surface generated by revolving the curve $y = \sqrt{x+1}$, $1 \le x \le 5$, about the x-axis. First sketch the graph.

5. The density function of a thin rod is $f(x) = (1 + x)^2$ on the interval $0 \le x \le 4$. Find the rod's moment to the origin and its center of mass.

6. Evaluate the limits.

$$\text{(a)}\ \lim_{x\to\infty} x^{\frac{1}{x}} \quad \text{(b)}\ \lim_{x\to 0} \frac{\arcsin x}{x} \quad \text{(c)}\ \lim_{x\to 0} \frac{\arctan x}{x}$$

7. Evaluate the integrals.

$$\text{(a)}\ \int_0^1 \frac{4x\,dx}{\sqrt{4-x^4}} \quad \text{(b)}\ \int_{\sqrt[4]{2}}^{\sqrt{2}} \frac{2x\,dx}{x^2\sqrt{x^4-1}} \quad \text{(c)}\ \int \tanh 2x\,dx$$

$$\text{(d)}\ \int_0^{\pi} \frac{\cos x\,dx}{1+\sin^2 x} \quad \text{(e)}\ \int x^2 \sin x\,dx \quad \text{(f)}\ \int_1^2 x \ln x\,dx$$

$$\text{(g)}\ \int e^x \sin x\,dx \quad \text{(h)}\ \int_0^{\pi/2} 3\sin^4 x \cos^3 x\,dx \quad \text{(i)}\ \int_0^{\pi/2} \sin 2x\ \cos 3x\,dx$$

$$\text{(j)}\ \int_{-2}^{2} \frac{dx}{x^2+4x+13} \quad \text{(k)}\ \int \frac{dx}{\sqrt{x^2-2x}} \quad \text{(l)}\ \int \frac{x+3}{2x^3-8x}\,dx$$

$$\text{(m)}\ \int_{-1}^{0} \frac{x^3-x}{(x^2+1)(x-1)^2}\,dx \quad \text{(n)}\ \int_0^{\pi/2} \frac{dx}{1+\sin x} \quad \text{(o)}\ \int \frac{2-e^x}{1+e^x}\,dx$$

8. Show the identities.

(a) $\sinh 2x = 2\sinh x \cosh x$ (b) $\cosh 2x = \cosh^2 x + \sinh^2 x$

(c) $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ (d) $\tanh^{-1} x = \dfrac{1}{2} \ln \dfrac{1+x}{1-x}, \quad |x| < 1$

9. Find the length of the curve $y = \cosh x$, $-2 \le x \le 2$.

10. (Minimal surface) Find the area of surface swept out by revolving the curve $y = 2\cosh(x/2)$, $0 \le x \le \ln 8$, about the x-axis.

11. (Hyperbolic functions and the hyperbola) Show that the variable u in the coordinates of $P(\cosh u, \sinh u)$ for the points of the right-hand branch of the hyperbola $x^2 - y^2 = 1$ is twice the area of the sector OAP, with $O(0,0)$ and $A(1,0)$, see Fig. 7.39 . (Hint: Find the area $A(u)$ as a function of u and show that $A'(u) = 1/2$ for any $u \in \mathbb{R}$. Then solve the corresponding initial value problem with initial condition $A(0) = 0$.)