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SEPARATUM

# ACTA MATHEMATICA
ACADEMIAE SCIENTIARUM HUNGARICAE

TOMUS XVI FASCICULI 3—4

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# REMARK ON A PAPER OF M. F. JANOWITZ

1965

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M. F. JANOWITZ has asked in his paper "A characterization of standard ideals" (*Acta Math. Acad. Sci. Hung.*, **16** (1965)) the following question:

Is every projective ideal of a lattice a homomorphism kernel?

In this note we give a counter-example to show that a projective ideal need not necessarily be a homomorphism kernel.

Consider the lattice on Fig. 1.

We assert that the ideal $(a]$ is a projective ideal, but it is not a homomorphism kernel. Indeed, an ideal $J$ is projective if and only if $a \in J$, $x \cap y \in J$ imply $(a \cup x) \cap y \in J$. (See in the paper of JANOWITZ, Theorem 3. 2). In our lattice $x \cap y \in (a]$, $(x, y \notin (a])$ if and only if $x = b$ and $c \leqq y \leqq g$ or $c = i$ (or symmetrically, interchanging $x$ and $y$), and so wet get $(a \cup x) \cap y \in (a]$, i. e. $(a]$ is projective.

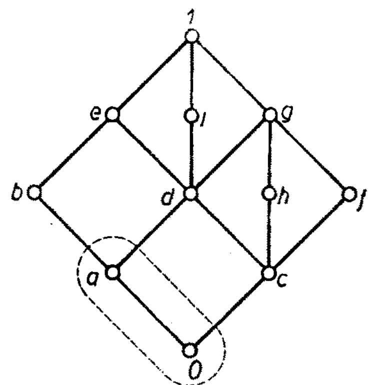


*Fig. 1*

Now we prove that $(a]$ is not a homomorphism kernel. Let us suppose that $a \equiv 0$ $(\Theta)$ for a suitable congruence relation. Then

$$f = (a \cup h) \cap f \equiv (0 \cup h) \cap f = c \ (\Theta)$$

and so

$$b = (f \cup i) \cap b \equiv (c \cup i) \cap b = a \ (\Theta),$$

$$\text{i. e. } b \equiv 0 \ (\Theta).$$

This is a contradiction if $(a]$ should be a homomorphism kernel.

Our counter-example and the definition of projective ideals suggest the following conjecture.

The usual definition of the homomorphism kernel is of second order type (using a familiar logical expression). The question nat. arises whether the notion of homomorphism kernel can be characterized with first order term? This means precisely the following. Does a formula $F$ of the first order logic with identity exist, such that $F$ contains as primitive non logical constants the lattice operations and a symbol $A$ for a subset of the universe and such that $F$ is true, in a lattice $L$ with a specified subset $L'$ as the interpretation for $A$ if and only if $L'$ is a homomorphism kernel of $L$? Our conjecture is that such a universal first order formula does not exist (a universal first order formula is formed from an open formula by prefixing to it universal quantities binding all the variables).

*(Received 16 March 1965)*