A)

$$f(x,y) = \begin{cases} \sin(y^2)/\sqrt{x^2+y^2} & \neq (0,0) \\ 0 & (0,0) \end{cases}$$

1)

Origón kívül:

$$f'_x(x,y) = \frac{-\sin(y^2) \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x}{x^2+y^2}$$

$$f'_y(x,y) = \frac{\cos(y^2) \cdot 2y \cdot \sqrt{x^2+y^2} - \sin(y^2) \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y}{x^2+y^2}$$

Origóban

$$f(0,y) = \frac{\sin y^2}{|y|} \qquad \lim_{y \to 0} \frac{\frac{\sin y^2}{|y|} - 0}{y} = \begin{cases} 1, & y > 0 \\ -1, & y < 0 \end{cases} \lightning$$

$f(x,0) = 0$ ($x=0$ esetén is 0) $\Rightarrow f'_x(0,0) = 0$

Az x-szerinti parc. derivált mindenhol létezik.
Az y-szerinti az origó kivételével mindenhol létezik.

Origó kivételével mindenhol tot. deriv. (parciálisok folytonosak). Origóban pedig nem parc. deriválható, így nem is tot. deriv.

2) $f(x,y) = x - \sin(xy)$

$f'_x(x,y) = 1 - \cos(xy)\cdot y$ $\qquad f'_x\left(1,\frac{\pi}{2}\right) = 1 - \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\cdot\frac{\pi}{2} = 1$

$f'_y(x,y) = -\cos(xy)\cdot x$ $\qquad f'_y\left(1,\frac{\pi}{2}\right) = -\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\cdot 1 = 0$

$f\left(1,\frac{\pi}{2}\right) = 1 - \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = 0$

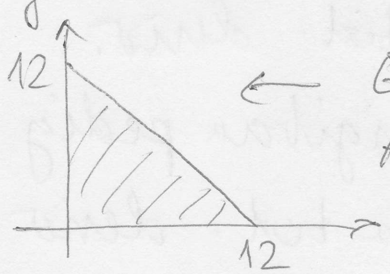
$\underline{P}\left(1,\frac{\pi}{2},0\right) \qquad \underline{n}(1,0,-1)$

$1(x-1) + 0\cdot\left(y-\frac{\pi}{2}\right) - (z-0) = 0$

$\underline{\underline{x - 1 - z = 0}}$

$\left|df\left(\left(1,\frac{\pi}{2}\right),(dx,dy)\right)\right| = |1\cdot dx + 0\,dy| = |dx| \le \underline{\underline{0{,}01}}$

3) $\left.\begin{array}{l} x+y+z=12 \\ xyz \to \max. \end{array}\right\}$ $f(x,y) = x\cdot y\cdot\underbrace{(12-x-y)}_{y \le -x+12}$



← Ezen kell f abs. maximuma.
A peremen $f \equiv 0$.
$(x,y \neq 0)$

$f'_x(x,y) = y\cdot(12-x-y) + xy(-1) = 0 \qquad 12-2x-y=0$

$f'_y(x,y) = x(12-x-y) + xy(-1) = 0 \qquad 12-x-2y=0$

$-x+y=0$

$x=y$

$12-3x=0 \quad \Leftarrow$

$x=4 \Rightarrow y=4$

$f(4,4) = 4\cdot4\cdot4 = \underline{64}$

Peremen 0, $(4,4)$-ben 64. Így a max. szorzat 64

$x=4,\ y=4,\ z=4$

4) $f(x,y) = x^3 + y^3 - 3xy$

$$\left.\begin{aligned} f'_x(x,y) &= 3x^2 - 3y = 0 \\ f'_y(x,y) &= 3y^2 - 3x = 0 \end{aligned}\right\} \qquad \begin{aligned} y &= x^2 \\ x &= y^2 \end{aligned} \qquad \begin{aligned} x^4 &= x \\ x(x^3-1) &= 0 \end{aligned}$$

Két stac. pont $\quad x=0 \quad x=1$
$\qquad\qquad\qquad\quad y=0 \quad y=1$

$$H = \begin{bmatrix} 6x & -3 \\ -3 & 6y \end{bmatrix}$$

$(0,0)$-ban $\begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix}$ $\quad D_1 = 0$, $D_2 = -9 < 0$

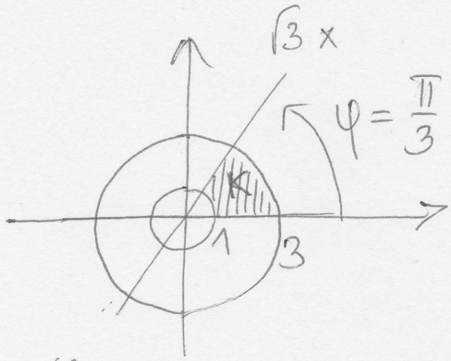
indef. $\Rightarrow$ nincs lok. sz.-é.

$(1,1)$-ben

$\begin{vmatrix} 6 & -3 \\ -3 & 6 \end{vmatrix}$ $\quad D_1 = 6 > 0$, $D_2 = 36 - 9 > 0$ $\quad$ poz. def.

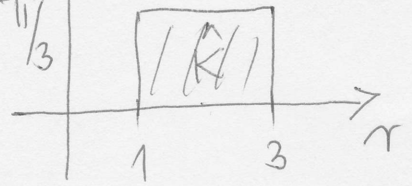
$(1,1)$ lok. minimum.

5)



$$\iint_K \operatorname{arctg} \frac{y}{x} \, dx\,dy =$$

$$= \iint_{\hat{K}} \underbrace{\operatorname{arctg} \underbrace{\frac{\cancel{r} \sin\varphi}{\cancel{r} \cos\varphi}}_{\operatorname{tg}\varphi}}_{\varphi} r\,dr\,d\varphi =$$



$$= \int_0^{\pi/3} \int_1^3 \varphi \cdot r \, dr \, d\varphi =$$

$$= \left(\int_0^{\pi/3} \varphi \, d\varphi\right) \cdot \left(\int_1^3 r \, dr\right) = \frac{(\pi/3)^2}{2} \cdot \left(\frac{3^2}{2} - \frac{1}{2}\right) = 2 \cdot (\pi/3)^2$$

$$\left[\frac{\varphi^2}{2}\right]_0^{\pi/3} \qquad \left[\frac{r^2}{2}\right]_1^3$$

A2

B)

1) $$f(x,y)=\begin{cases}\dfrac{\sin(x^2)}{\sqrt{x^2+y^2}} & \neq(0,0)\\ 0 & (0,0)\end{cases}$$

Origón kívül:

$$f'_x(x,y)=\frac{\cos(x^2)\cdot 2x\sqrt{x^2+y^2}-\sin(x^2)\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x}{x^2+y^2}$$

$$f'_y(x,y)=\frac{-\sin(x^2)\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2y}{x^2+y^2}$$

Origóban:

$f(0,y)=0$ (origóban is) $\quad f'_y(0,0)=0$

$$f(x,0)=\frac{\sin x^2}{|x|}\qquad \lim_{x\to 0}\frac{\frac{\sin x^2}{|x|}-0}{x}=\begin{cases}1, \text{ ha } x>0\\ -1, \text{ ha } x<0\end{cases} \lightning$$

Parciálisan:

x-szerint az origón kívül mindenhol
y-szerint mindenhol deriválható.

Totálisan:

origón kívül mindenhol (folyt. parc. der)
origóban nem tot. der., mert nem parc. deriválható

2) $f(x,y) = x - \cos(xy)$ $\qquad (x_0, y_0) = (0, \pi)$

$f'_x(x,y) = 1 + \sin(xy)\cdot y \qquad f'_x(0,\pi) = 1 + \sin(0\pi)\pi = 1$

$f'_y(x,y) = \sin(xy)\cdot x \qquad f'_y(0,\pi) = \sin(0\cdot\pi)\cdot 0 = 0$

$f(0,\pi) = 0 - \cos(\underbrace{0\cdot\pi}_{0}) = -1$

érintősík: $\underline{P}(0, \pi, -1) \quad \underline{n}(1, 0, -1)$

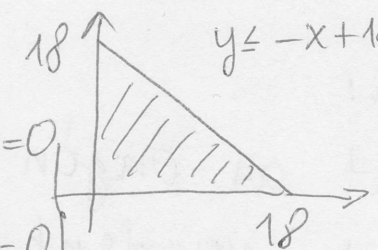
$1(x-0) + 0(y-\pi) - (z+1) = 0$

$\underline{\underline{x - z - 1 = 0}}$

3) $|df((0,\pi),(dx,dy))| = |1\cdot dx + 0dy| = |dx| \le \underline{\underline{0{,}02}}$

$x + y + z = 18$ $\qquad f(x,y) = x\cdot y\cdot(18 - x - y)$ abs. max. kell

$xyz \to \max$



$y \le -x + 18$

peremen nulla!

$f'_x(x,y) = y(18-x-y) + xy(-1) = 0$

$\underline{f'_y(x,y) = x(18-x-y) + xy(-1) = 0}$

$x, y \neq 0$

$$\left.\begin{aligned} 18 - 2x - y &= 0 \\ \underline{18 - x - 2y} &= 0 \end{aligned}\right] -$$

$-x + y = 0$

$x = y \Rightarrow 18 - 3x = 0 \Rightarrow x = 6$

$\Downarrow$

$y = 6$

$\Downarrow$

$z = 6$

$f(6,6) = 6^3$

peremen nulla, $(6,6)$-ban 6^3

Így abs. max. $(6,6)$-ban, értéke 6^3

4) $f(x,y) = 3xy - x^3 - y^3$

$$\left.\begin{aligned} f'_x(x,y) &= 3y - 3x^2 = 0 \\ f'_y(x,y) &= 3x - 3y^2 = 0 \end{aligned}\right\} \quad \left.\begin{aligned} y &= x^2 \\ x &= y^2 \end{aligned}\right\} \Rightarrow \begin{aligned} &x = x^4 \\ &x(1-x^3) = 0 \end{aligned}$$

Két stac. pont: $\begin{array}{ll} x=0 & x=1 \\ y=0 & y=1 \end{array}$

$$H = \begin{bmatrix} -6x & 3 \\ 3 & -6y \end{bmatrix}$$

(0,0)-ban:

$$\begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} \qquad \begin{aligned} D_1 &= 0 \\ D_2 &= -9 < 0 \end{aligned}$$

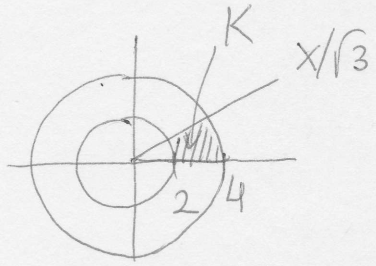
indef., nincs lok. sz. é.

(1,1)-ben

$$\begin{vmatrix} -6 & 3 \\ 3 & -6 \end{vmatrix} \qquad \begin{aligned} D_1 &= -6 < 0 \\ D_2 &= 36 - 9 > 0 \end{aligned}$$

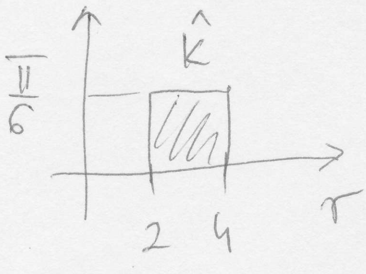
neg. def., azaz lokális max.

5)



$$\iint_K \operatorname{arctg}\frac{y}{x}\,dx\,dy = \iint_{\hat{K}} \operatorname{arctg}\frac{\cancel{r}\sin\varphi}{\cancel{r}\cos\varphi}\, r\,dr\,d\varphi$$

$$= \iint_{\hat{K}} r\varphi\,dr\,d\varphi = \int_0^{\pi/6}\int_2^4 r\varphi\,dr\,d\varphi =$$



$$= \left[\frac{r^2}{2}\right]_2^4 \cdot \left[\frac{\varphi^2}{2}\right]_0^{\pi/6} =$$

$$= \left(\frac{4^2}{2} - \frac{2^2}{2}\right) \cdot \frac{(\pi/6)^2}{2} = 3(\pi/6)^2$$