Ⓐ 1) $y' = 2\sqrt{y+1}\cdot\cos x \quad , \; y(\pi) = 0$

$\int \frac{1}{\sqrt{y+1}}\,dy = 2\int \cos x\,dx$

szétválasztható.

$y \neq -1$, de ez úgysem megoldása a kezdetiért. feladatnak

$\int (y+1)^{-1/2}\,dy = 2\int \cos x\,dx$

$\frac{(y+1)^{1/2}}{1/2} = 2\sin x + C \quad / \cdot \frac{1}{2}$

$\sqrt{y+1} = \sin x + \frac{C}{2}$

$\underbrace{\sqrt{0+1}}_{1} = \underbrace{\sin\pi}_{0} + \frac{C}{2}$

$\Downarrow$

$\frac{C}{2} = 1$

$\sqrt{y+1} = \sin x + 1 \quad \Leftarrow$

$(y+1) = (\sin x + 1)^2$

$\underline{\underline{y(x) = \sin^2 x + 2\sin x}}$

2) $u = y^3 \Rightarrow u' = 3y^2\cdot y'$

$y' + 2y = \frac{x}{y^2} \quad / \cdot 3y^2$

$\underbrace{3y^2y'}_{u'} + 6\underbrace{y^3}_{u} = 3x$

$u' + \underbrace{6}_{p(x)}u = \underbrace{3x}_{q(x)} \quad$ OK.

$\mu(x) = e^{\int 6\,dx} = e^{6x}$

$u(x) = e^{-6x}\Big(\underbrace{\int 3x e^{6x}\,dx}_{\parallel} + C\Big) \Rightarrow$

$\int \frac{3}{6}x\,(e^{6x})'\,dx = \frac{1}{2}xe^{6x} - \int \frac{1}{2}e^{6x}\,dx \Rightarrow$

$\frac{1}{2}xe^{6x} - \frac{1}{12}e^{6x}$

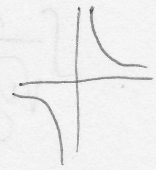
$Ce^{-6x} + \frac{1}{2}x - \frac{1}{12} = u(x)$

$\Uparrow$

$y(x) = \sqrt[3]{Ce^{-6x} + \frac{1}{2}x - \frac{1}{12}}$

3) $y' = xy^3$

a) $xy^3 = 1$ $\quad (1,1) \quad (-1,-1)$ pl. a két izoklinára esik $\quad y = \frac{1}{\sqrt[3]{x}}$



b) $y' = xy^3 \qquad 0 \cdot 2^3 = 0$ lehet szélsőérték

$y'' = y^3 + x3y^2 \cdot y'$

$2^3 + 0 \cdot 3 \cdot 2^2 \cdot 0 = 8 > 0 \Rightarrow$ lok. min. van a 0-ban.

4) $y'' + 3y' = 5$ $\quad$ H $\quad \lambda^2 + 3\lambda = 0$

$\lambda(\lambda + 3) = 0 \Rightarrow y_{há}(x) = C_1 + C_2 e^{-3x}$

IH külső rez.

$y_{ip}(x) = A \cdot x$

$y'_{ip}(x) = A$

$y''_{ip}(x) = 0$

$0 + 3 \cdot A = 5 \Rightarrow A = \frac{5}{3}$

$$\underline{y(x) = C_1 + C_2 e^{-3x} + \frac{5}{3}x}$$

5) $f_n = f_{n-1} + 2f_{n-2} \Rightarrow q^2 - q - 2 = 0$

$\Leftarrow (q-2)(q+1) = 0$

$f_n = C_1 2^n + C_2(-1)^n$

$f_0 = C_1 + C_2 = 2$

$f_1 = 2C_1 - C_2 = -2$

$3C_1 = 0 \Rightarrow C_1 = 0$

$C_2 = 2$

$f_n = 2(-1)^n$

$f_{1000} = 2 \cdot (-1)^{1000} = \underline{2}$

(B) $y' = 2\sqrt{y-1}\cdot\cos x \quad y(\pi)=2$

1) $\int \frac{1}{\sqrt{y-1}}\,dy = \int 2\cos x\,dx$

$\int (y-1)^{-1/2}\,dy = 2\sin x + C$

$\frac{(y-1)^{1/2}}{1/2} = 2\sin x + C$

$\sqrt{y-1} = \sin x + \frac{C}{2} \qquad \underbrace{\sqrt{2-1}}_{1} = \underbrace{\sin\pi}_{0} + \frac{C}{2} \Downarrow \frac{C}{2} = 1$

$\sqrt{y-1} = \sin x + 1$

$\underline{\underline{y(x) = \sin^2 x + 2\sin x + 2}}$

2) $y' + y = \frac{2x}{y^2} \;/\cdot 3y^2 \qquad u = y^3 \Rightarrow u' = 3y^2y'$

$\underbrace{3y^2y'}_{u'} + 3\underbrace{y^3}_{u} = 6x$

$u' + \underbrace{3}_{p(x)}u = \underbrace{6x}_{q(x)}$ ok.

$\mu(x) = e^{\int 3\,dx} = e^{3x}$

$u(x) = e^{-3x}\Big(\underbrace{\int 6x e^{3x}\,dx}_{\|} + c\Big)$

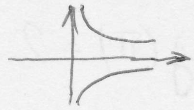
$\int 2x(e^{3x})'\,dx = 2xe^{3x} - \int 2e^{3x}\,dx =$

$= 2xe^{3x} - \frac{2}{3}e^{3x}$

$u(x) = ce^{-3x} + 2x - \frac{2}{3}$

$\underline{\underline{y(x) = \sqrt[3]{Ce^{-3x} + 2x - \frac{2}{3}}}}$

3) $y' = xy^2$



$y = \pm\frac{1}{\sqrt{x}},\ x>0$

a) $xy^2 = 1$ $\quad (1,1)\quad \left(4, \frac{1}{2}\right)$ ilyen pontok

b) $y(0) = 1$

$y' = xy^2 \qquad 0\cdot 1^2 = 0$ lehet szélsőérték

$y'' = y^2 + x2yy'$

$1^2 + 0\cdot 2\cdot 1\cdot 0 = 1 > 0$ lok. minimum van 0-ban.

4) $y'' + 2y' = 7$

(H) $\lambda^2 + 2\lambda = 0$

$\lambda(\lambda+2) = 0 \Rightarrow y_{há}(x) = c_1 + c_2 e^{-2x}$

(IH) Külső rez.

$y_{ip}(x) = Ax$

$y'_{ip}(x) = A$

$y''_{ip}(x) = 0$

$0 + 2\cdot A = 7 \Rightarrow A = \frac{7}{2}$

$$y_{iá}(x) = c_1 + c_2 e^{-2x} + \frac{7}{2}x$$

5) $f_n + 3f_{n-1} + 2f_{n-2} = 0$

$q^2 + 3q + 2 = 0$

$(q+1)(q+2) = 0 \Rightarrow f_n = c_1(-1)^n + c_2(-2)^n$

$f_0 = c_1 + c_2 = 3$

$f_1 = -c_1 - 2c_2 = -3$

$-c_2 = 0 \Rightarrow c_2 = 0$

$c_1 = 3$

$f_n = 3(-1)^n$

$f_{1000} = 3\cdot(-1)^{1000} = 3$