AZ MEGOLDÁSOK 2016 JANUÁR 5.

(1) a) $\sum_{n=2}^{\infty} \frac{2^{2n-1}}{8^n} = \sum_{n=2}^{\infty} \frac{1}{2} \cdot \frac{4^n}{8^n} = \sum_{n=2}^{\infty} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^n =$

$= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{16} + \ldots =$ GEOMETRIAI SOR

$\boxed{a = \frac{1}{8},\ q = \frac{1}{2}}$, ÍGY a) $= \frac{1}{8} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{1}{4}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot e^n - e^{2n}}{e^{3n}} = \sum_{n=0}^{\infty} (-e^{-2})^n - \sum_{n=0}^{\infty} (e^{-1})^n =$

$=$ GEOMETRIAI SOROK $|q| < 1$ $= \frac{1}{1 - (-e^{-2})} - \frac{1}{1 - e^{-1}}$

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(2) a) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

$$\frac{\sin(2x^2)}{x} = \frac{1}{x} \cdot \left(2x^2 - \frac{(2x^2)^3}{3!} + \frac{(2x^2)^5}{5!} - \frac{(2x^2)^7}{7!} + \ldots \right)$$

$$= 2x - \frac{2^3}{3!} \cdot x^5 + \frac{2^5}{5!} \cdot x^9 - \frac{2^7}{7!} \cdot x^{13} + \ldots$$

$\boxed{c_0 = 0}$ $\boxed{c_1 = 2}$ $\boxed{c_2 = c_3 = c_4 = 0}$ $\boxed{c_5 = -\frac{2^3}{3!}}$

b) $\int_0^{1/2} \frac{\sin(2x^2)}{x}\, dx = 2 \cdot \int_0^{1/2} x\, dx - \frac{2^3}{3!} \int_0^{1/2} x^5\, dx + \frac{2^5}{5!} \int_0^{1/2} x^9\, dx -$

$- \frac{2^7}{7!} \int_0^{1/2} x^{13}\, dx + \ldots = 2 \cdot \frac{(1/2)^2}{2} - \frac{2^3}{3!} \frac{(1/2)^6}{6} + \frac{2^5}{5!} \frac{(1/2)^{10}}{10} - \frac{2^7}{7!} \cdot \frac{1}{14} \cdot (1/2)^{14} + \ldots =$

$$= \frac{1}{4} - \frac{1}{3!}\cdot\frac{1}{2^3}\cdot\frac{1}{6} + \frac{1}{5!}\cdot\frac{1}{2^5}\cdot\frac{1}{10} - \frac{1}{7!}\cdot\frac{1}{2^7}\cdot\frac{1}{14} + \dots$$

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3) $$\int_0^{2\pi} \sin(kx)\cdot\sin(lx)\,dx = \int_0^{2\pi} \frac{1}{2}\cdot\Big(\cos\big((k-l)\cdot x\big) - \cos\big((k+l)\cdot x\big)\Big)\,dx = ☆$$

HA $k=0$ VAGY $l=0$ AKKOR ☆ $=0$ ✓

HA $k\geq 1$ ÉS $l\geq 1$ ÉS $k=l$, AKKOR

$$☆ = \int_0^{2\pi} \frac{1}{2}\cdot\big(1-\cos(2kx)\big)\,dx = [\dots] = \pi$$

$\boxed{\text{LÁSD SZKENNELT 43. OLDAL}}$

HA $k\geq 1$ ÉS $l\geq 1$ ÉS $k\neq l$, AKKOR

$$☆ = \frac{1}{2}\int_0^{2\pi} \cos\big((k-l)\cdot x\big)\,dx - \frac{1}{2}\int_0^{2\pi}\cos\big((k+l)\cdot x\big)\,dx = [\dots] = 0$$

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4) a) LÁSD SZKENNELT JEGYZET 60. OLDAL:

$\boxed{x=2}$ $\boxed{y=1}$ $\boxed{z=3}$

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b) LÁS 80. OLDAL, 82. OLDAL

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5) LÁSD 126, 127, 128, 129. OLDAL

SAJÁTÉRTÉKEK: ± 1

$$\underline{P} = \begin{bmatrix} 0 & -\frac{2}{\sqrt5} & \frac{1}{\sqrt5} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \end{bmatrix} \qquad \underline{P}^T\cdot\underline{A}\cdot\underline{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

⑥ LÁSD 141. OLDAL.

$$g(x,y) = 1 \cdot \ln(xy) + x \cdot \frac{1}{xy} \cdot y = \ln(xy) + 1$$

$$h(x,y) = x \cdot \frac{1}{xy} \cdot x = \frac{x}{y}$$

$$\left.\begin{aligned} \frac{\partial}{\partial x} h(x,y) &= \frac{1}{y} \\ \frac{\partial}{\partial y} g(x,y) &= \frac{1}{xy} \cdot x = \frac{1}{y} \end{aligned}\right\}$$ VALÓBAN EGYENLŐEK (YOUNG-TÉTEL)

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⑦ KELL: $x^2+y^2=9$

$$\frac{\partial}{\partial x}\left(x^2+2y^2+\lambda\cdot(x^2+y^2-9)\right) = 2x + \lambda\cdot 2x = 0$$

$$\frac{\partial}{\partial y}\left(x^2+2y^2+\lambda\cdot(x^2+y^2-9)\right) = 4y + \lambda\cdot 2y = 0$$

HA $x \neq 0$, AKKOR $\lambda = -\frac{2x}{2x} = -1$, ÍGY

$4y - 2y = 0 \Rightarrow y = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

TEHÁT $(3,0)$ ÉS $(-3,0)$ LAGRANGE-KRIT. PONT

HA $y \neq 0$, AKKOR $\lambda = -\frac{4y}{2y} = -2$, ÍGY

$2x - 4x = 0 \Rightarrow x = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$

TEHÁT $(0,3)$ ÉS $(0,-3)$ LAGRANGE-KRIT PONT

$f(3,0) = f(-3,0) = 9$ MINIMUM

$f(0,3) = f(0,-3) = 2 \cdot 9$ MAXIMUM

8) $\boxed{0 \le x \le 1}$ $\boxed{0 \le y \le 1-x}$

$$0 \le \frac{1}{2} z \le 1-x-y \Rightarrow \boxed{0 \le z \le 2-2x-2y}$$

TEHÁT $V = \int_{x=0}^{1} \int_{y=0}^{1-x} (2-2x-2y)\, dy\, dx =$

$$= 2 \cdot \int_0^1 \int_0^{1-x} (1-x-y)\, dy\, dx = 2 \cdot \int_0^1 \left[y - xy - \frac{y^2}{2} \right]_{y=0}^{y=1-x} dx =$$

$$= 2 \cdot \int_0^1 \left((1-x) - x \cdot (1-x) - \frac{(1-x)^2}{2} \right) dx = 2 \cdot \int_0^1 (1-x) \cdot \left(1 - x - \frac{1-x}{2}\right) dx =$$

$$= \int_0^1 (1-x)^2\, dx = \left[-\frac{(1-x)^3}{3} \right]_0^1 = (-0) - \left(-\frac{1}{3}\right) = \frac{1}{3}$$

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9) $\iiint_D f(x,y,z)\, dx\, dy\, dz = \int_{\alpha=0}^{\pi} \int_{\beta=0}^{2\pi} \int_{r=0}^{10} r \cdot r^2 \cdot \sin(\alpha)\, dr\, d\beta\, d\alpha$

GÖMBI KOORD

$$= \int_{\alpha=0}^{\pi} \int_{\beta=0}^{2\pi} \left[\frac{r^4}{4} \right]_0^{10} \cdot \sin(\alpha)\, d\beta\, d\alpha =$$

$$= 2500 \cdot 2\pi \cdot \int_0^{\pi} \sin(\alpha)\, d\alpha = 10000 \cdot \pi$$