AZ DEC 21 VIZSGA MEGO:

1. a) LÁSD SZKENNELT 9. OLDAL

b) $$\underbrace{\sum_{n=1}^{\infty} \frac{1}{n^{2/3}+n}}_{} \geq \sum_{n=1}^{\infty} \frac{1}{n+n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = +\infty$$

$\boxed{\text{MINORÁNS KRIT}}$ $\boxed{\text{HARMONIKUS SOR}}$

TEHÁT $\to$ EZ A SOR DIVERGENS.

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2. $\sum_{n=0}^{\infty} \left(\frac{-(x+4)}{3} \right)^n$ GEOMETRIAI SOR: $\sum_{n=0}^{\infty} q^n$

KONVERGENS, HA $|q|<1$, AZAZ $\left| \frac{-(x+4)}{3} \right| < 1$, AZAZ

HA $\boxed{-7 < x < -1}$. KONV. SUGÁR: $\boxed{R = 3}$

KONV. INTERVALLUM: $(-7, -1)$ (NYÍLT)

HA $\boxed{-7 < x - 1}$, AKKOR $f(x) = \frac{1}{1-q} = \frac{1}{1+\frac{x+4}{3}} =$

$= \frac{3}{x+7}$

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3. $\sin^2(x) + \sin^3(x) = \frac{1-\cos(2x)}{2} + \frac{1-\cos(2x)}{2} \cdot \sin(x) =$

$= \frac{1}{2} - \frac{1}{2}\cos(2x) + \frac{1}{2}\sin(x) - \frac{1}{2}\sin(x) \cdot \cos(2x)$

1. OLDAL

$$= \frac{1}{2} - \frac{1}{2}\cos(2x) + \frac{1}{2}\sin(x) - \frac{1}{4}\cdot\left(\sin(3x) + \sin(-x)\right)$$

$$= \frac{1}{2} + \frac{3}{4}\sin(x) - \frac{1}{2}\cos(2x) - \frac{1}{4}\sin(3x)$$

$$\boxed{p(x) = \frac{1}{2} + \frac{3}{4}\sin(x)} \qquad \int_0^{2\pi} \left(f(x) - p(x)\right)^2 dx =$$

$$= \int_0^{2\pi} \left(-\frac{1}{2}\cos(2x) - \frac{1}{4}\sin(3x)\right)^2 dx = \pi\cdot\left(\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{4}\right)^2\right)$$

$$= \frac{5}{16}\pi$$

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4 a) $W = \text{lin}(\underline{v}_1, \ldots, \underline{v}_r) = \{\underline{v} : \underline{v} = \lambda_1 \cdot \underline{v}_1 + \ldots + \lambda_r \cdot \underline{v}_r\}$

HA $\underline{v}_1, \ldots, \underline{v}_r$ LINEÁRISAN FÜGGETLENEK,
AKKOR $B = \{\underline{v}_1, \ldots, \underline{v}_r\}$ BÁZISA W-NEK

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b) $x + 2y + 3z = 0$: y ÉS z SZABAD VÁLTOZÓK:

$$\boxed{x = -2y - 3z} \; : \; \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2y - 3z \\ y \\ z \end{pmatrix} = y \cdot \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + z \cdot \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

TEHÁT $B = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$ BÁZISA

V-NEK.

5 a) $\underline{\underline{P}}_{B',B} = \begin{bmatrix}1&1\\0&1\end{bmatrix}$ (HISZ: OSZOPAI: $\underline{b}_1$ ÉS $\underline{b}_2$)

b) $\underline{\underline{P}}_{B,B'} = \left(\underline{\underline{P}}_{B',B}\right)^{-1} = \begin{bmatrix}1&-1\\0&1\end{bmatrix}$

c) $\underline{\underline{T}}_B = \begin{bmatrix}0&1\\-1&0\end{bmatrix}$ (HISZ: OSZLOPAI: $T(\underline{e}_1)$ ÉS $T(\underline{e}_2)$)

d)
$$\left.\begin{aligned} T(\underline{b}_1) &= T(\underline{e}_1) = \begin{pmatrix}0\\-1\end{pmatrix} = \underline{b}_1 - \underline{b}_2 \\ T(\underline{b}_2) &= T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} = 2\underline{b}_1 - \underline{b}_2 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow \underline{\underline{T}}_{B'} = \begin{bmatrix}1&2\\-1&-1\end{bmatrix}$ UGYANEZ MÁSFÉLEKÉPP:

$$\underline{\underline{T}}_{B'} = \underline{\underline{P}}_{B,B'} \cdot \underline{\underline{T}}_B \cdot \underline{\underline{P}}_{B',B} = \begin{bmatrix}1&-1\\0&1\end{bmatrix} \cdot \begin{bmatrix}0&1\\-1&0\end{bmatrix} \cdot \begin{bmatrix}1&1\\0&1\end{bmatrix} = \begin{bmatrix}1&2\\-1&-1\end{bmatrix}$$

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6 $f_x = y \cdot e^{x-2y}$ || $f_y = e^{x-2y} - 2y \cdot e^{x-2y}$

$f_x(1,-1) = -e^3$ || $f_y(1,-1) = e^3 - 2\cdot(-1)\cdot e^3 = 3e^3$

$\|4\underline{i} - 3\underline{j}\| = \sqrt{4^2+(-3)^2} = \sqrt{25} = 5$

a) $= \left\langle \left(\frac{4}{5}, \frac{-3}{5}\right), \left(-e^3, 3e^3\right) \right\rangle = -\frac{4}{5}\cdot e^3 - \frac{3}{5}\cdot 3\cdot e^3 = -\frac{13}{5}e^3$

b) LEGGYORS. NÖV: $\begin{pmatrix}-1/\sqrt{10}\\ 3/\sqrt{10}\end{pmatrix}$ LEGGYORS. CSÖKK: $\begin{pmatrix}1/\sqrt{10}\\ -3/\sqrt{10}\end{pmatrix}$

7 a) $\left.\begin{array}{l} f_x = 2x - 4y + 2 \\ f_y = -4x + 4y \end{array}\right\}$ $\boxed{\begin{array}{l} 2x - 4y + 2 = 0 \\ -4x + 4y = 0 \end{array}}$ $\Rightarrow$ $\boxed{x = y}$

$\Rightarrow 2x - 4x + 2 = 0 \Rightarrow \boxed{x = 1}$ $\boxed{y = 1}$

EGYETLEN STACIONÁRIUS PONT: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\boxed{\text{HESSE MÁTRIX}}$: $\begin{array}{|c|c|} \hline f_{xx} & f_{xy} \\ \hline f_{yx} & f_{yy} \\ \hline \end{array} = \begin{array}{|c|c|} \hline 2 & -4 \\ \hline -4 & 4 \\ \hline \end{array} \leftarrow \det = 2 \cdot 4 - (-4)^2 < 0$

TEHÁT $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ NYEREGPONT.

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b) $\boxed{y = 2x + 2} \Rightarrow f(x, 2x+2) =$

$= x^2 - 4x \cdot (2x+2) + 2 \cdot (2x+2)^2 + 2x + 5 =$

$= x^2 - 8x^2 - 8x + 8x^2 + 16x + 8 + 2x + 5 =$

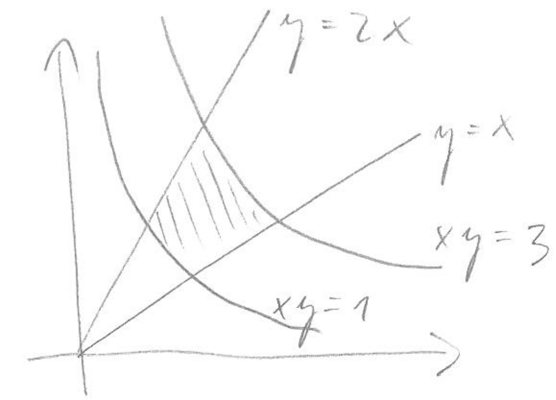
$= x^2 + 10x + 13 = g(x) \quad g'(x) = 2x + 10$

$\Rightarrow g'(-5) = 0, \; g''(x) = 2$ KONVEX FÜGGV.

$\Rightarrow \min_x g(x) = g(-5) = (-5)^2 + 10 \cdot (-5) + 13$

$= -12$

8)



$\boxed{\dfrac{y}{x} = u}$ $\boxed{1 \le u \le 2}$

$\boxed{xy = v}$ $\boxed{1 \le v \le 3}$

$$u \cdot v = \frac{y}{x} \cdot xy = y^2 \Rightarrow \boxed{y = \sqrt{u} \cdot \sqrt{v}}$$

$$v/u = (xy) \cdot \frac{x}{y} = x^2 \Rightarrow \boxed{x = \frac{1}{\sqrt{u}} \cdot \sqrt{v}}$$

$\underline{\text{JACOBI-MÁTRIX}}$:

$$\underline{J}(u,v) = \begin{array}{|c|c|} \hline x_u & x_v \\ \hline y_u & y_v \\ \hline \end{array} = \begin{array}{|c|c|} \hline -\frac{1}{2} u^{-3/2} \cdot v^{1/2} & \frac{1}{2} \cdot u^{-1/2} \cdot v^{-1/2} \\ \hline \frac{1}{2} \cdot u^{-1/2} \cdot v^{1/2} & \frac{1}{2} u^{1/2} \cdot v^{-1/2} \\ \hline \end{array}$$

$$\det\left(\underline{J}(u,v)\right) = -\frac{1}{4} \cdot u^{-1} - \frac{1}{4} \cdot u^{-1} = -\frac{1}{2} \cdot u^{-1}$$

$$\text{TERÜLET} = \int_{u=1}^{2} \int_{v=1}^{3} \frac{1}{2} \cdot \frac{1}{u} \, dv \, du = \int_{1}^{2} \frac{1}{u} \, du = \ln(2)$$

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9) HENGER KOORD:

$$M = \int_{\varphi=\pi/4}^{\pi/2} \int_{r=1}^{2} \int_{z=-1}^{1} \frac{z^2 \cdot r}{r^4} \, dz \, dr \, d\varphi =$$

$$= \int_{\varphi=\pi/4}^{\pi/2} \int_{r=1}^{2} \frac{2}{3} \frac{1}{r^3} \, dr \, d\varphi = \frac{2}{3} \cdot \int_{\pi/4}^{\pi/2} \left[-\frac{1}{2} \frac{1}{r^2}\right]_1^2 d\varphi =$$

$$= \frac{2}{3} \cdot \int_{\pi/4}^{\pi/2} \left(-\frac{1}{2} \frac{1}{4} + \frac{1}{2} \frac{1}{1}\right) d\varphi = \frac{2}{3} \cdot \frac{3}{8} \cdot \left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{16}$$