C CSOPORT:

1.
$$\begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} = (-1)\cdot \begin{vmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} =$$

$$= (-1)\cdot \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} = (-1)\cdot \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 4 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 6 \end{vmatrix} = 6$$

---

2. a) LÁSD SZKENNELT JEGYZET 96. OLD ÉS 93. OLDAL.

b) $$\underline{v} = \underbrace{\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}}_{\underline{\underline{A}}\cdot\underline{\alpha}} = \underbrace{\begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix} \cdot \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}}_{\underline{\underline{B}}\cdot\underline{\beta}} \Rightarrow \underline{\beta} = (\underline{\underline{B}})^{-1}\cdot \underline{\underline{A}}\cdot\underline{\alpha}$$

$$\boxed{\underline{\underline{B}}^{-1} = ?} \quad \left(\begin{matrix} -2 & 3 \\ 1 & -2 \end{matrix}\middle|\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\right) \rightarrow \left(\begin{matrix} 1 & -2 \\ -2 & 3 \end{matrix}\middle|\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\right) \rightarrow$$

$$\left(\begin{array}{cc|cc} 1 & -2 & 0 & 1 \\ 0 & -1 & 1 & 2 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & -2 & 0 & 1 \\ 0 & 1 & -1 & -2 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -2 & -3 \\ 0 & 1 & -1 & -2 \end{array}\right)$$

$$(\underline{\underline{B}})^{-1} = \begin{pmatrix} -2 & -3 \\ -1 & -2 \end{pmatrix} \qquad (\underline{\underline{B}})^{-1} \cdot \underline{\underline{A}} =$$

			3	4
			-2	-3
-2	-3		0	1
-1	-2		1	2

$$\underline{\beta} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \cdot \underline{\alpha} \Rightarrow \boxed{\begin{array}{l} \beta_1 = \alpha_2 \\ \beta_2 = \alpha_1 + 2\alpha_2 \end{array}}$$

---

3) a) ELŐSZÖR ORTOGONALIZÁLUNK:

$$\underline{w}_1 = \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\underline{w}_2 = \underline{v}_2 - \alpha \cdot \underline{w}_1 \quad , \quad \langle \underline{w}_1, \underline{w}_2 \rangle = 0 \Rightarrow \alpha = \frac{\langle \underline{w}_1, \underline{v}_2 \rangle}{\langle \underline{w}_1, \underline{w}_1 \rangle}$$

$$\alpha = 1\cdot(-1) + 1\cdot 1 + 1 \cdot 0 = 0 \Rightarrow \underline{w}_2 = \underline{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\underline{w}_3 = \underline{v}_3 - \beta \cdot \underline{w}_1 - \gamma \cdot \underline{w}_2 \quad \beta = \frac{\langle \underline{w}_1, \underline{v}_3 \rangle}{\langle \underline{w}_1, \underline{w}_1 \rangle} = \frac{4}{3}$$

$$\gamma = \frac{\langle \underline{w}_2, \underline{v}_3 \rangle}{\langle \underline{w}_2, \underline{w}_2 \rangle} = \frac{1}{2}$$

$$\underline{w}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \frac{4}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/6 \\ 1/6 \\ -1/3 \end{pmatrix}$$

AZTÁN ORTONORMÁLUNK:

2. OLDAL

$$\underline{u}_1 = \frac{\underline{w}_1}{\|\underline{w}_1\|} = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \qquad \underline{u}_2 = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\underline{u}_3 = \frac{\underline{w}_3}{\|\underline{w}_3\|} = \begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{pmatrix}$$

b) $\underline{\underline{A}} = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{pmatrix}$ $\underline{\underline{A}}$ ORTOGONÁLIS,

ÍGY $\underline{\underline{A}}^{-1} = \underline{\underline{A}}^{T} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{pmatrix}$

---

4 a) $\det(\underline{\underline{A}} - \lambda \cdot I_2) = (3-\lambda)\cdot(-3-\lambda) - (-4)\cdot 2 =$

$= (\lambda^2 - 9) + 8 = \lambda^2 - 1 \Rightarrow \boxed{\lambda_1 = -1} \quad \boxed{\lambda_2 = 1}$

$\boxed{\lambda_1 = -1}$: $\left.\begin{aligned} (3+1)\cdot x - 4\cdot y &= 0 \\ 2\cdot x + (-3+1)\cdot y &= 0 \end{aligned}\right\}$ $\boxed{x = y} \Rightarrow \boxed{\underline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

$\boxed{\lambda_2 = 1}$: $\left.\begin{aligned} (3-1)\cdot x - 4\cdot y &= 0 \\ 2\cdot x + (-3-1)\cdot y &= 0 \end{aligned}\right\}$ $\boxed{x = 2y} \Rightarrow \boxed{\underline{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$

$\underline{\underline{Q}} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \qquad \underline{\underline{D}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

④ b) $\underline{\underline{D}} = \underline{\underline{Q}}^{-1} \cdot \underline{\underline{A}} \cdot \underline{\underline{Q}}$ $\qquad$ $\underline{\underline{A}} = \underline{\underline{Q}} \cdot \underline{\underline{D}} \cdot \underline{\underline{Q}}^{-1}$

$\underline{\underline{A}}^{2015} = \underline{\underline{Q}} \cdot \underline{\underline{D}}^{2015} \cdot \underline{\underline{Q}}^{-1}$ $\qquad$ $\underline{\underline{D}}^{2015} = \begin{pmatrix} (-1)^{2015} & 0 \\ 0 & 1^{2015} \end{pmatrix} = \underline{\underline{D}}$

$\underline{\underline{A}}^{2015} = \underline{\underline{Q}} \cdot \underline{\underline{D}} \cdot \underline{\underline{Q}}^{-1} = \underline{\underline{A}} = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}$
