A CSOPORT:

1) a) LÁSD SZKENNELT JEGYZET 82. OLDAL.

b) $\underline{\underline{A}}\,\underline{x} = \underline{b}$ $\quad \underline{\underline{A}} = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$$\det(\underline{\underline{A}}) = 1\cdot 2\cdot(-1) + (-1)\cdot 1\cdot 2 + 1\cdot(-1)\cdot(-1)$$
$$- 1\cdot 2\cdot 2 - (-1)\cdot(-1)\cdot(-1) - 1\cdot 1\cdot(-1) =$$
$$= -2 -2 +1 -4 +1 +1 = -5$$

HELYETTESÍTSÜK $\underline{b}$-VEL $\underline{\underline{A}}$ MÁSODIK OSZLOPÁT:

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ 2 & -1 & -1 \end{vmatrix} = 1\cdot 2\cdot(-1) + 1\cdot 1\cdot 2 + 1\cdot(-1)\cdot(-1)$$
$$- 1\cdot 2\cdot 2 - 1\cdot(-1)\cdot(-1) - 1\cdot 1\cdot(-1) =$$
$$= -2 + 2 + 1 - 4 - 1 + 1 = -3$$

TEHÁT $\boxed{y = \frac{-3}{-5} = \frac{3}{5}}$

2) a) $\underline{\underline{A}}^T = \begin{pmatrix} 1 & 0 & 2 & 3 \\ -3 & 3 & -3 & -3 \\ 2 & 6 & -2 & 6 \\ 0 & -6 & 6 & 0 \end{pmatrix}$ EZEN HAJTUNK VÉGRE ELEMI SORMŰVELETEKET.

$$\rightarrow \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 3 & 3 & 6 \\ 0 & 6 & -6 & 0 \\ 0 & -6 & 6 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ REDUKÁLT LÉPCSŐS ALAK

OSZLOPTÉR EGY BÁZISA:

$$\underline{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix} \quad \underline{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad \underline{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$\underline{b}$) (SORTÉR DIMENZIÓJA) = (MÁTRIX RANGJA) =
= (OSZLOPTÉR DIMENZIÓJA) = 3

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③ a) S NORMÁLVEKTORA: $\underline{n} = (2, -1, 0)$

HA $\underline{x} \in \mathbb{R}^3$, AKKOR $T(\underline{x}) = (\underline{x}$ VETÜLETE S-RE)

KELL: $\boxed{T(\underline{x}) \perp \underline{n}}$ $\boxed{T(\underline{x}) = \underline{x} - \lambda \cdot \underline{n}}$ KELL: $\boxed{\langle T(\underline{x}), \underline{n} \rangle = 0}$

$$\langle \underline{x} - \lambda \cdot \underline{n}, \underline{n} \rangle = \langle \underline{x}, \underline{n} \rangle - \lambda \cdot \langle \underline{n}, \underline{n} \rangle = 0 \Rightarrow$$

$$\Rightarrow \boxed{\lambda = \frac{\langle \underline{x}, \underline{n} \rangle}{\langle \underline{n}, \underline{n} \rangle}} = \frac{2 \cdot x - y}{2^2 + (-1)^2 + 0^2} = \frac{2x - y}{5}$$

$$(\underline{\underline{B}} \text{ ELSŐ OSZLOPA}) = T(\underline{e}_1) = \underline{e}_1 - \lambda \cdot \underline{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \frac{2}{5} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} =$$

$$\boxed{= \begin{pmatrix} 1/5 \\ 2/5 \\ 0 \end{pmatrix}} \quad (\underline{\underline{B}} \text{ MÁSODIK OSZLOPA}) = T(\underline{e}_2) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \frac{-1}{5} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} =$$

$$\boxed{= \begin{pmatrix} 2/5 \\ 4/5 \\ 0 \end{pmatrix}} \quad (\underline{\underline{B}} \text{ HARMADIK OSZL.}) = T(\underline{e}_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \frac{0}{5} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

2. OLDAL

TEHÁT: $\underline{\underline{B}} = \begin{pmatrix} 1/5 & 2/5 & 0 \\ 2/5 & 4/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $T(T(\underline{x})) = T(\underline{x})$ (A VETÍTETT KÉP VETÜLETE ÖNMAGA), TEHÁT $\underline{\underline{B}}^2 = \underline{\underline{B}}$,

$\underline{\underline{B}}^3 = \underline{\underline{B}}$, ..., $\boxed{\underline{\underline{B}}^{100} = \underline{\underline{B}}}$ $\boxed{\text{SÍK}}$

rang$(\underline{\underline{B}})$ = (KÉPTÉR DIMENZIÓJA) = 2

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4 a) $5x^2 + 2y^2 + 4xy = (x, y) \cdot \underline{\underline{A}} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$, $\underline{\underline{A}} = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$

$\det(\underline{\underline{A}} - \lambda \cdot \underline{\underline{I}}_2) = (5-\lambda)\cdot(2-\lambda) - 4 = \lambda^2 - 7\lambda + 6 = 0$

$\lambda_{1,2} = \dfrac{7 \pm \sqrt{49 - 4\cdot 6}}{2} = \dfrac{7 \pm 5}{2}$ $\to \boxed{\lambda_1 = 6}$, $\to \boxed{\lambda_2 = 1}$

$\boxed{\lambda_1 = 6}$: $\left.\begin{aligned} (5-6)\cdot x + 2\cdot y &= 0 \\ 2\cdot x + (2-6)\cdot y &= 0 \end{aligned}\right\} \Rightarrow \boxed{x = 2y} \Rightarrow \underline{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\boxed{\lambda_2 = 1}$: $\left.\begin{aligned} (5-1)\cdot x + 2y &= 0 \\ 2x + (2-1)\cdot y &= 0 \end{aligned}\right\} \Rightarrow \boxed{2x = -y} \Rightarrow \underline{v}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

ORTONORMÁLT SAJÁTBÁZIS: $\underline{w}_1 = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$, $\underline{w}_2 = \begin{pmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{pmatrix}$

TEHÁT $\underline{\underline{P}}^T \cdot \underline{\underline{A}} \cdot \underline{\underline{P}} = \begin{array}{|c|c|} \hline 6 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$ $\underline{\underline{P}} = \begin{pmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix}$

3. OLDAL

VAGY MINT $\begin{pmatrix} x \\ y \end{pmatrix} = \underline{\underline{P}} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix}$, ÍGY $\underline{\underline{P}}^{-1} = \underline{\underline{P}}^T$ MIATT

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underline{\underline{P}}^T \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$, TEHÁT

$$\boxed{x' = \frac{2}{\sqrt{5}} \cdot x + \frac{1}{\sqrt{5}} \cdot y} \qquad \boxed{y' = \frac{-1}{\sqrt{5}} \cdot x + \frac{2}{\sqrt{5}} \cdot y}$$

$$\boxed{5x^2 + 2y^2 + 4xy = 6 \cdot (x')^2 + 1 \cdot (y')^2}$$