AZ, 2 ZH PÓTLÁSA:

1 a) LÁSD SZKENNELT JEGYZET
82. OLDALA

$$\underline{A} \cdot \mathrm{adj}(\underline{A}) = \det(\underline{A}) \cdot I_n = \begin{pmatrix} \det(\underline{A}) & 0 & 0 & 0 \\ 0 & \det(\underline{A}) & 0 & 0 \\ 0 & 0 & \det(\underline{A}) & 0 \\ 0 & 0 & 0 & \det(\underline{A}) \end{pmatrix}$$

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1 b) HARMADIK OSZLOP SZERINT KIFEJTVE:

$$\det(\underline{A}) = (-1)^{3+4} \cdot (-1) \cdot \begin{vmatrix} -2 & 1 & -1 \\ 1 & 2 & 0 \\ -2 & 1 & 1 \end{vmatrix} =$$

$$= (-2) \cdot 2 \cdot 1 + 1 \cdot 0 \cdot (-2) + (-1) \cdot 1 \cdot 1$$

$$- (-1) \cdot 2 \cdot (-2) - 1 \cdot 1 \cdot 1 - (-2) \cdot 0 \cdot 1 =$$

$$= -4 - 1 - 4 - 1 = -10$$

2

$$\left(\begin{array}{ccccc|c} 2 & 6 & 0 & 4 & 18 & 0 \\ 2 & 6 & -5 & 4 & -2 & 0 \\ 1 & 3 & -2 & 2 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 9 & 0 \\ 2 & 6 & -5 & 4 & -2 & 0 \\ 1 & 3 & -2 & 2 & 0 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 9 & 0 \\ 0 & 0 & -5 & 0 & -20 & 0 \\ 0 & 0 & -2 & 0 & -9 & 0 \end{array}\right) \to \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 9 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & -2 & 0 & -9 & 0 \end{array}\right) \to$$

$$\rightarrow \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 9 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 9 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

x_2 ÉS x_4 SZABAD VÁLTOZÓK.

$\boxed{x_5 = 0}$ $\boxed{x_3 = 0}$ $x_1 + 3x_2 + 2x_4 + 9 \cdot 0 = 0$

$$\Rightarrow \boxed{x_1 = -3x_2 - 2x_4} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -3x_2 - 2x_4 \\ x_2 \\ 0 \\ x_4 \\ 0 \end{pmatrix} =$$

$$= x_2 \cdot \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \cdot \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

TEHÁT

$$B = \left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

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3 a) $T(\underline{b}_1) = -\underline{b}_2$, $T(\underline{b}_2) = \underline{b}_1$, TEHÁT

$$T_B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

b) $T(\underline{b}'_1) = 2 \cdot \underline{b}_2$, $T(\underline{b}'_2) = -\frac{1}{2} \cdot \underline{b}'_1$, ÍGY

$$T_{B'} = \begin{pmatrix} 0 & -1/2 \\ 2 & 0 \end{pmatrix}$$

2. OLDAL

④ $\det\left(\begin{array}{|c|c|} \hline -3/5-\lambda & -4/5 \\ \hline -4/5 & 3/5-\lambda \\ \hline \end{array}\right) = \lambda^2 - \frac{9}{25} - \frac{16}{25} = \lambda^2 - 1$

SAJÁTÉRTÉKEK: $\boxed{\lambda_1 = 1}$, $\boxed{\lambda_2 = -1}$

SAJÁTVEKTOROK:

$$\boxed{\lambda_1 = 1} \Rightarrow \left.\begin{array}{l} x\cdot(-3/5-1) + y\cdot(-4/5) = 0 \\ x\cdot(-4/5) + y\cdot(3/5-1) = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -\frac{8}{5}\cdot x - \frac{4}{5}y = 0 \\ -\frac{4}{5}x - \frac{2}{5}y = 0 \end{array}\right\} \Rightarrow \boxed{y = -2x} \Rightarrow \underline{w}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\boxed{\lambda_2 = -1} \quad \left.\begin{array}{l} x\cdot(-3/5+1) - 4/5\cdot y = 0 \\ x\cdot(-4/5) + (3/5+1)y = 0 \end{array}\right\} \Rightarrow \boxed{x = 2y} \quad \underline{w}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

ORTONORMÁLT SAJÁTVEKTOROK:

$$\left.\begin{array}{l} \boxed{\lambda_1 = 1} : \underline{v}_1 = \dfrac{\underline{w}_1}{\|\underline{w}_1\|} = \begin{pmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{pmatrix} \\ \boxed{\lambda_2 = -1} : \underline{v}_1 = \dfrac{\underline{w}_2}{\|\underline{w}_2\|} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix} \end{array}\right\}$$

ORTONORMÁLT SAJÁT BÁZIS