D CSOPORT:

① a) LÁSD JEGYZET 8. OLDAL.

b) HA $X \geq 2$, AKKOR X ÉS $\ln(X)$ IS POZITÍV ÉS NÖVEKVŐ, ÍGY $f(X) = \dfrac{1}{X \cdot \ln^2(X)}$ POZITÍV ÉS CSÖKKENŐ FÜGGVÉNY. ELÉG TEHÁT ELDÖNTENI, HOGY $\int\limits_2^\infty \dfrac{1}{X \cdot \ln^2(x)} dX$ KONVERGENS-E.

$$\frac{d}{dX}\left(\frac{1}{\ln(x)}\right) = \frac{-1}{\ln^2(x)} \cdot \frac{1}{X}, \text{ ÍGY } \int\limits_2^b \frac{1}{X \cdot \ln^2(X)} dX =$$

$$= \left[\frac{-1}{\ln(X)}\right]_2^b = \frac{-1}{\ln(b)} - \frac{-1}{\ln(2)} = \frac{1}{\ln(2)} - \frac{1}{\ln(b)}$$

TEHÁT $\lim\limits_{b \to \infty} \int\limits_2^b \dfrac{1}{X \cdot \ln^2(X)} dX = \lim\limits_{b \to \infty} \left(\dfrac{1}{\ln(2)} - \dfrac{1}{\ln(b)}\right) =$

$$= \frac{1}{\ln(2)} - \frac{1}{\lim\limits_{b \to \infty} \ln(b)} = \frac{1}{\ln(2)} - \frac{1}{\infty} = \frac{1}{\ln(2)} < +\infty$$

TEHÁT $\int\limits_2^\infty \dfrac{1}{X \cdot \ln^2(X)} dX$ KONVERGENS, ÉS ÍGY $\sum\limits_{n=2}^\infty \dfrac{1}{n \cdot \ln^2(n)}$ IS <u>KONVERGENS.</u>

D CSOPORT:

② a) $e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$

$$e^{-x^3} = 1 - x^3 + \frac{x^6}{2} - \frac{x^9}{6} + \dots$$

$$x \cdot e^{-x^3} = x - x^4 + \frac{x^7}{2} - \frac{x^{10}}{6} + \dots$$

TEHÁT $x \cdot e^{-x^3} = \sum_{n=0}^{\infty} c_n \cdot x^n$, AHOL

$\boxed{c_0 = 0}$ $\boxed{c_1 = 1}$ $\boxed{c_2 = 0}$ $\boxed{c_3 = 0}$ $\boxed{c_4 = -1}$

TUDJUK: $c_n = \frac{1}{n!} f^{(n)}(0)$, AZAZ $f^{(n)}(0) = n! \cdot c_n$

ÍGY $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = 0$, $f^{(4)}(0) = -24$

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③ a) $\sin^2(x) \cdot \cos(x) + \cos^2(x) = \left(\frac{1 - \cos(2x)}{2}\right) \cdot \cos(x) + \left(\frac{1 + \cos(2x)}{2}\right)$

$$= \frac{1}{2}\cos(x) - \frac{1}{2} \cdot \left(\cos(2x) \cdot \cos(x)\right) + \frac{1}{2} + \frac{1}{2}\cos(2x)$$

$$= \frac{1}{2}\cos(x) - \frac{1}{2} \cdot \left(\frac{1}{2} \cdot \left(\cos(3x) + \cos(x)\right)\right) + \frac{1}{2} + \frac{1}{2} \cdot \cos(2x) =$$

$$= \frac{1}{2} + \frac{1}{4} \cdot \cos(x) + \frac{1}{2}\cos(2x) - \frac{1}{4}\cos(3x)$$, TEHÁT

$\boxed{a_0 = \frac{1}{2}}$, $\boxed{a_1 = \frac{1}{4}}$ $\boxed{a_2 = \frac{1}{2}}$ $\boxed{a_3 = -\frac{1}{4}}$ $\boxed{a_k = 0}$, HA $k \geq 4$, $\boxed{b_k = 0}$

2. OLDAL

③ f) $f(x) = \sin^2(x) \cdot \cos(x) + \cos^2(x)$ PÁROS FÜGGVÉNY,

ÍGY $f^2(x)$ IS PÁROS, ÍGY

$$\int_0^{\pi} f^2(x)\,dx = \frac{1}{2}\int_{-\pi}^{\pi} f^2(x)\,dx = \frac{1}{2}\int_0^{2\pi} f^2(x)\,dx = ☆$$

HISZEN $f^2(x)$ 2π-PERIODIKUS.

$$☆ = \frac{1}{2}\cdot\left(2\pi\cdot a_0^2 + \pi\cdot a_1^2 + \pi\cdot a_2^2 + \pi\cdot a_3^2\right) =$$

$$= \frac{1}{2}\cdot\left(2\pi\cdot\frac{1}{4} + \pi\cdot\frac{1}{16} + \pi\cdot\frac{1}{4} + \pi\cdot\frac{1}{16}\right) =$$

$$= \left(\frac{\pi}{4} + \frac{\pi}{32} + \frac{\pi}{8} + \frac{\pi}{32}\right) = \pi\cdot\frac{8+1+4+1}{32} = \pi\cdot\frac{7}{16}$$

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④
$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}\right)
\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & -3 & -1 & -3 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & -3 & -1 & -3 & 0 & 1 \end{array}\right)
\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 1/2 & -3 & -3/2 & 1 \end{array}\right)
\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -6 & -3 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 7 & 3 & -2 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & -6 & -3 & 2 \end{array}\right)
\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & -6 & -3 & 2 \end{array}\right)
\underline{\underline{A^{-1}}} = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 1 & -1 \\ -6 & -3 & 2 \end{pmatrix}$$