C CSOPORT:

(1) a) HA $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = q$ ÉS

$\boxed{q<1}$, AKKOR $\sum_{n=1}^{\infty} a_n$ KONVERGENS

$\boxed{q>1}$, AKKOR $\sum_{n=1}^{\infty} a_n$ DIVERGENS

$\boxed{q=1}$, AKKOR NEM SEGÍT A HÁNYADOS-KRIT.

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b) $a_n = \frac{n!}{n^n}$ $\quad \frac{|a_{n+1}|}{|a_n|} = \frac{(n+1)!/n!}{(n+1)^{n+1}/n^n} =$

$$= \frac{n+1}{(n+1)\cdot\left(\frac{n+1}{n}\right)^n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$$

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} = q$$

$\boxed{\frac{1}{e}<1}$, TEHÁT $q<1$, TEHÁT $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ KONVERGENS

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(2) a) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots$$

$$\frac{\sin(x^2)}{x^2} = 1 - \frac{x^4}{3!} + \frac{x^8}{5!} - \frac{x^{12}}{7!} + \ldots$$

C CSOPORT:

② b) $\int_0^2 \frac{\sin(x^2)}{x^2}\,dx = \int_0^2 \left(1 - \frac{x^4}{3!} + \frac{x^8}{5!} - \frac{x^{12}}{7!} + \dots\right)dx =$

$= \left[x - \frac{x^5}{5\cdot 3!} + \frac{x^9}{9\cdot 5!} - \frac{x^{13}}{13\cdot 7!} + \dots\right]_0^2 =$

$= 2 - \frac{2^5}{5\cdot 3!} + \frac{2^9}{9\cdot 5!} - \frac{2^{13}}{13\cdot 7!} + \dots$

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③ a) PÁROS FÜGGVÉNY, ÍGY TISZTA KOSZINUSZOS A FOURIER-SORA: $a_0 + \sum_{k=1}^{\infty} a_k \cdot \cos(kx)$

$a_0 = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 2\,dx = \frac{1}{2\pi} \cdot 2 \cdot \pi = 1$

$a_k = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \cos(kx)\,dx = \frac{2}{\pi} \cdot \int_0^{\pi} f(x) \cdot \cos(kx)\,dx =$

$= \frac{2}{\pi} \cdot \int_0^{\pi/2} 2 \cdot \cos(kx)\,dx = \frac{4}{\pi} \cdot \left[\frac{\sin(kx)}{k}\right]_0^{\pi/2} = \frac{4 \cdot \sin\left(\frac{k\pi}{2}\right)}{\pi \cdot k}$

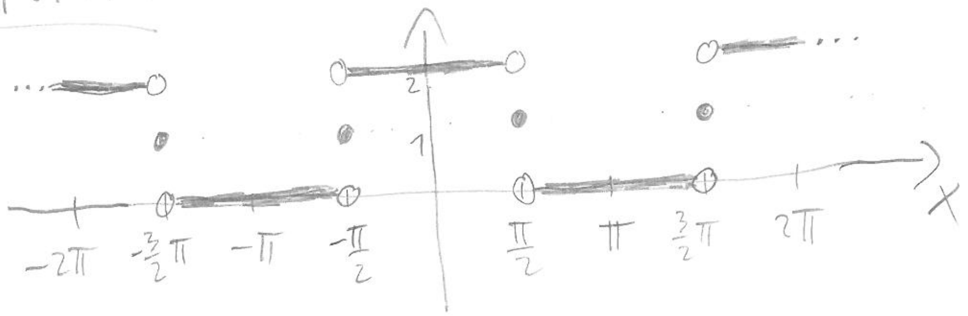
$\sin\left(\frac{k\cdot\pi}{2}\right) = \begin{cases} 0, \text{ HA } k = 2,4,6,8,\dots \\ 1, \text{ HA } k = 1,5,9,13,\dots \\ -1, \text{ HA } k = 3,7,11,15,\dots \end{cases}$ TEHÁT

$p(x) = a_0 + \sum_{k=1}^{4} a_k \cdot \cos(kx) = 1 + \frac{4}{\pi} \cdot \cos(x) - \frac{4}{\pi \cdot 3} \cdot \cos(3x)$

HISZEN $\boxed{a_0 = 1}$, $\boxed{a_1 = \frac{4}{\pi}}$, $\boxed{a_2 = 0}$, $\boxed{a_3 = -\frac{4}{\pi\cdot 3}}$, $\boxed{a_4 = 0}$

C CSOPORT:

(3) b)



(4)

$$\left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 1 & 1 & -3 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 0 & -2 & 3 & -1 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \\ 0 & -2 & 3 & -1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & -2 & 3 & -1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & -6 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 & 2 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 7 & -6 & 12 \\ 0 & 1 & 0 & 2 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 & 2 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 & 2 \end{array}\right)$$

TEHÁT $\underline{\underline{A}}^{-1} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -2 & 3 \\ 1 & -1 & 2 \end{pmatrix}$