AZ 1ZH PÓTLÁSA:

① a) ABSZOLÚT KONV., HA $\sum_{n=1}^{\infty} |a_n|$ KONVERGENS

FELTÉTELESEN KONV., HA $\sum_{n=1}^{\infty} a_n$ KONV., DE $\sum_{n=1}^{\infty} |a_n|$ DIVERGENS.

b) $\boxed{a_n = (-1)^n \cdot \frac{\ln(n)}{(n+1)^2}}$ $\boxed{|a_n| = \frac{\ln(n)}{(n+1)^2}, \text{ HA } n \geq 2}$

$$\ln(n) \leq n \Rightarrow \ln(\sqrt{n}) \leq \sqrt{n} \Rightarrow \ln(n) \leq 2 \cdot \sqrt{n}$$

$$\Rightarrow \frac{\ln(n)}{(n+1)^2} \leq \frac{2 \cdot \sqrt{n}}{(n+1)^2} \leq \frac{2 \cdot \sqrt{n}}{n^2} = 2 \cdot n^{-3/2}$$

$$\Rightarrow \sum_{n=2}^{\infty} |a_n| \leq \sum_{n=2}^{\infty} 2 \cdot n^{-3/2} = 2 \cdot \sum_{n=2}^{\infty} n^{-\alpha} \underset{\boxed{\alpha > 1}}{<} +\infty$$

TEHÁT $\sum_{n=1}^{\infty} \frac{\ln(n)}{(n+1)^2}$ ABSZOLÚT KONVERGENS

2) $a_n = \dfrac{(4x-5)^n}{\sqrt{n^2+3}}$ $\qquad \sum_{n=0}^{\infty} a_n$ KONV. VAGY DIV.?

GYÖK-KRITÉRIUM: $\lim_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} =$

$$= \lim_{n\to\infty} |4x-5| \cdot \frac{\sqrt{n^2+3}}{\sqrt{(n+1)^2+3}} = |4x-5| = q$$

$$\boxed{q<1} \iff |4x-5| < 1 \iff \left|x - \frac{5}{4}\right| < \frac{1}{4} \iff$$

$$x \in \left(\frac{5}{4} - \frac{1}{4}, \frac{5}{4} + \frac{1}{4}\right) \iff \boxed{x \in \left(1, \frac{3}{2}\right)}$$

KONVERGENCIA-SUGÁR: $\boxed{R = 1/4}$

KONV. INTERVALLUM VÉGPONTJAI ?

$$\boxed{x=1} \quad \sum_{n=0}^{\infty} \frac{(4\cdot 1-5)^n}{\sqrt{n^2+3}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}} = \star$$

$\boxed{b_n = \dfrac{1}{\sqrt{n^2+3}}}$ $\quad b_0 \geq b_1 \geq b_2 \geq \ldots$ MONOTON CSÖKK

$\boxed{\lim_{n\to\infty} b_n = 0}$ LEIBNIZ-KRIT $\Rightarrow \star$ KONV.

$$\boxed{x = 3/2} \Rightarrow \sum_{n=0}^{\infty} \frac{(4\cdot 3/2 - 5)^n}{\sqrt{n^2+3}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+3}} \geq \sum_{n=0}^{\infty} \frac{1}{\sqrt{(n+3)^2}} =$$

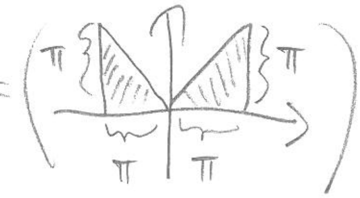
$$= \sum_{n=0}^{\infty} \frac{1}{n+3} = +\infty$$

DIV.

TEHÁT: KONV. INTERVALL: $[1, 3/2)$

2,0C DAC

③ f PÁROS $\Rightarrow$ TISZTA KOSZINUSZOS A FOURIER-SORA.

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\, dx = \Bigg($  $\Bigg) \cdot \frac{1}{2\pi} = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$

$\boxed{k \geq 1}$: $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |x| \cdot \cos(kx)\, dx = \frac{2}{\pi}\int_{0}^{\pi} x \cdot \cos(kx)\, dx =$

$$= \underbrace{\frac{2}{\pi} \cdot \left[x \cdot \frac{\sin(kx)}{k} \right]_0^{\pi}}_{0} - \frac{2}{\pi}\int_0^{\pi} \frac{\sin(kx)}{k}\, dx =$$

$$= \frac{2}{\pi}\left[\frac{\cos(kx)}{k^2} \right]_0^{\pi} = \frac{2}{\pi} \cdot \frac{(-1)^k - 1}{k^2}$$

TEHÁT $p(x) = a_0 + a_1 \cdot \cos(x) + a_2 \cdot \cos(2x) + a_3 \cdot \cos(3x) =$

$$= \frac{\pi}{2} + \frac{2}{\pi} \cdot \frac{-2}{1^2} \cdot \cos(x) + \frac{2}{\pi} \cdot \frac{0}{2^2} + \frac{2}{\pi} \cdot \frac{-2}{3^2} \cdot \cos(3x)$$

$$= \frac{\pi}{2} - \frac{4}{\pi} \cdot \cos(x) - \frac{4}{\pi} \cdot \frac{1}{9} \cdot \cos(3x)$$

(4) $\left(\begin{array}{ccc|c} 1 & 2 & -3 & -8 \\ 3 & -1 & 5 & 4 \\ 4 & 1 & a & b+4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -3 & -8 \\ 0 & -7 & 14 & 28 \\ 0 & -7 & a+12 & b+36 \end{array}\right) \to$

$\to \left(\begin{array}{ccc|c} 1 & 2 & -3 & -8 \\ 0 & 1 & -2 & -4 \\ 0 & -7 & a+12 & b+36 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -3 & -8 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & a-2 & b+8 \end{array}\right)$

TEHÁT HA $\boxed{a \neq 2}$ AKKOR PONTOSAN EGY MEGOLDÁS VAN.

HA $\boxed{a=2}$ ÉS $\boxed{b \neq -8}$ AKKOR NINCS MEGOLDÁS.

HA $\boxed{a=2}$ ÉS $\boxed{b=-8}$ AKKOR VÉGTELEN SOK MEGOLDÁS VAN.