C CSOPORT

① (a) SZKENNELT JEGYZET 69. OLDAL

(b) 70. OLDAL

(c) HA $\boxed{g(x) = x}$, AKKOR $\dfrac{f(b)-f(a)}{g(b)-g(a)} = \dfrac{f(b)-f(a)}{b-a}$

ÉS $\dfrac{f'(x_0)}{g'(x_0)} = f'(x_0)$, ÍGY ADÓDIK LAGRANGE

---

② $$\frac{n-10}{\sqrt{3n}+n} = \frac{n+\sqrt{3n}}{n+\sqrt{3n}} + \left(-\frac{\sqrt{3n}+10}{n+\sqrt{3n}}\right) = 1 + a_n$$

$$\boxed{b_n = \sqrt{2n}+1} \qquad \lim_{n\to\infty} a_n \cdot b_n = -\lim_{n\to\infty} \frac{\sqrt{3n}+10}{n+\sqrt{3n}} \cdot \left(\sqrt{2n}+1\right) = -\sqrt{6}$$

JOLLY JOKER KÉPLET (LÁSD 29. OLDAL)

$$\lim_{n\to\infty} (1+a_n)^{b_n} = \exp(-\sqrt{6}) = e^{-\sqrt{6}}$$

---

③ a) $$\frac{(x^3 \cdot \ln(x))' \cdot (x+1) \cdot \sin(x) - x^3 \cdot \ln(x) \cdot ((x+1) \cdot \sin(x))'}{((x+1) \cdot \sin(x))^2} =$$

$$= \frac{\left(3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x}\right) \cdot (x+1) \cdot \sin(x) - x^3 \cdot \ln(x) \cdot \left(1 \cdot \sin(x) + (x+1) \cdot \cos(x)\right)}{(x+1)^2 \cdot \sin^2(x)}$$

③ b) $f(x) = \cos(x)^{ch(x)} = \exp\left(ch(x) \cdot \ln(\cos(x))\right)$

$$f'(x) = \exp'\left(ch(x) \cdot \ln(\cos(x))\right) \cdot \left(ch(x) \cdot \ln(\cos(x))\right)' =$$

$$= \cos(x)^{ch(x)} \cdot \left(sh(x) \cdot \ln(\cos(x)) + ch(x) \cdot \frac{-\sin(x)}{\cos(x)}\right)$$

---

④ $\lim\limits_{x \to 0-} f(x) = B \cdot e^{-0} + \tan(0) = B$

$\boxed{y = 3x}$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{1}{\sin(3x)} - \frac{1}{3x} =$$

$$= \lim_{y \to 0+} \frac{1}{\sin(y)} - \frac{1}{y} = 0$$

$\boxed{\text{LÁSD 74-75. OLDAL}}$

TEHÁT $\boxed{B = 0}$

---

⑤ $f(x) = (x+1) \cdot e^{-x}$ $\quad f'(x) = 1 \cdot e^{-x} + (x+1) \cdot (-e^{-x}) =$

$\boxed{= -x \cdot e^{-x}}$ $\quad f''(x) = \frac{d}{dx}\left(-x \cdot e^{-x}\right) = -1 \cdot e^{-x} - x \cdot (-e^{-x}) =$

$= (x-1) \cdot e^{-x}$

f NÖVŐ $\iff f' > 0 \iff -x > 0 \iff x \in (-\infty, 0)$

f CSÖKKENŐ $\iff f' < 0 \iff -x < 0 \iff x \in (0, \infty)$

f KONVEX $\iff f'' > 0 \iff x - 1 > 0 \iff x \in (1, \infty)$

f KONKÁV $\iff f'' < 0 \iff x - 1 < 0 \iff x \in (-\infty, 1)$