B CSOPORT (SZKENNELT JEGYZET)

(1) a) 82. OLDAL b) 84. OLDAL

c) $f(x) = x^4$, $x_0 = 0$, 84. OLDAL

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(2) $$\frac{3n-\sqrt{2n}}{3n+1} = \frac{3n+1}{3n+1} + \frac{\sqrt{2n}-1}{3n+1} = 1 + a_n$$

$\boxed{\text{JOLLY JOKER}}$ (29. OLDAL): $b_n = \sqrt{2n}-1$

$$\lim_{n\to\infty} a_n \cdot b_n = \frac{\sqrt{2n}-1}{3n+1} \cdot (\sqrt{2n}-1) = \frac{2}{3}$$

$$\lim_{n\to\infty} (1+a_n)^{b_n} = \exp\left(2/3\right) = e^{2/3}$$

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(3) a) $$\frac{d}{dx} \ln(\operatorname{sh}(x^2+3x)) =$$

$$\ln'(\operatorname{sh}(x^2+3x)) \cdot \operatorname{sh}'(x^2+3x) \cdot (x^2+3x)' =$$

$$= \frac{1}{\operatorname{sh}(x^2+3x)} \cdot \operatorname{ch}(x^2+3x) \cdot (2x+3) =$$

$$= \operatorname{cth}(x^2+3x) \cdot (2x+3) = \frac{2x+3}{\operatorname{th}(x^2+3x)}$$

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(3) b) $$f(x) = (\ln(x))^{\sqrt[3]{x}} = \exp(x^{1/3} \cdot \ln(\ln(x)))$$

$$f'(x) = f(x) \cdot (x^{1/3} \cdot \ln(\ln(x)))' =$$

$$= (\ln(x))^{\sqrt[3]{x}} \cdot \left(\frac{1}{3} \cdot x^{-2/3} \cdot \ln(\ln(x)) + x^{1/3} \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x}\right)$$

④ $\lim\limits_{x\to 0_-} f(x) = \operatorname{sh}(0) + A\cdot \operatorname{ch}(0) = A$

$$\lim_{x\to 0_+} \frac{\arctan(x) - x}{x^3} \overset{\frac{0}{0},\, L'H}{=\!=} \lim_{x\to 0_+} \frac{\frac{1}{1+x^2} - 1}{3x^2} =$$

$$= \lim_{x\to 0_+} \frac{\frac{1}{1+x^2} - \frac{1+x^2}{1+x^2}}{3x^2} = \lim_{x\to 0_+} \frac{\left(\frac{-x^2}{1+x^2}\right)}{3x^2} = \lim_{x\to 0_+} \left(-\frac{1}{3}\right)\cdot \frac{1}{1+x^2} = \frac{-1}{3}$$

LEGYEN HÁT $\boxed{A = -\frac{1}{3}}$, AMÚGY AZ f FOLYT.

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⑤ $f'(x) = 6x - 3x^2 = 3\cdot x\cdot(2-x)$

KRITIKUS PONTOK: $f'(x) = 0 \iff \boxed{x = 0}\ \boxed{x = 2}$

$f(-3) = 3\cdot(-3)^2 - (-3)^3 = 27 - (-27) = \underbrace{-54}$

GLOB. MIN

$f(0) = 0$

$f(2) = 3\cdot 2^2 - 2^3 = 12 - 8 = 4 \leftarrow$ GLOB. MAX

$f(3) = 3\cdot 3^2 - 3^3 = 27 - 27 = 0$