A1, 1. ZH, 2015 MÁRC 6., A CSOPORT

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(1) $z_1 \cdot z_2 = (r_1 \cdot r_2) \cdot \left(\cos(\varphi_1 + \varphi_2) + i \cdot \sin(\varphi_1 + \varphi_2) \right)$

$1/z_1 = \left(\frac{1}{r_1}\right) \cdot \left(\cos(-\varphi_1) + i \cdot \sin(-\varphi_1) \right)$

$z_2^{10} = (r_2)^{10} \cdot \left(\cos(10 \cdot \varphi_2) + i \cdot \sin(10 \cdot \varphi_2) \right)$

$z_2 \cdot \overline{z_2} = (r_2)^2 \cdot \left(\cos(0) + i \cdot \sin(0) \right)$

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(2) a) $\dfrac{z_1}{z_2} = \dfrac{z_1 \cdot \overline{z_2}}{|z_2|^2} = \dfrac{(-3+3i)\cdot(2\cdot\sqrt{3}+2i)}{(2\cdot\sqrt{3})^2 + 2^2} =$

$= \dfrac{-6\cdot\sqrt{3} - 6 + 6\cdot\sqrt{3}\cdot i - 6i}{16}$

$= -\dfrac{3}{8}\cdot(\sqrt{3}+1) + \dfrac{3}{8}\cdot(\sqrt{3}-1)\cdot i$

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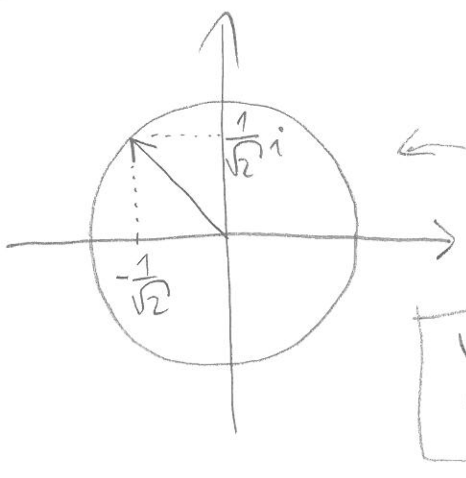
b)

$|z_1| = \sqrt{(-3)^2 + 3^2} = 3\cdot\sqrt{2} = r_1$



$\dfrac{z_1}{|z_1|} = -\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\cdot i$

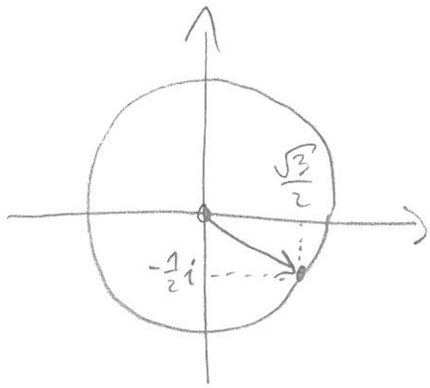
$\boxed{\varphi_1 = \dfrac{3}{4}\pi}$

1. OLDAL

TEHÁT $z_1 = (3\cdot\sqrt{2})\cdot\left(\cos\left(\frac{3}{4}\pi\right)+i\cdot\sin\left(\frac{3}{4}\pi\right)\right)$

$$|z_2| = \sqrt{(-2)^2+(2\cdot\sqrt{3})^2} = 4 = r_1$$

$$\frac{z_2}{|z_2|} = \frac{\sqrt{3}}{2} - \frac{1}{2}\cdot i \longrightarrow$$



$$\boxed{\varphi_2 = -\frac{\pi}{6}\left(= \frac{11\pi}{6}\right)}$$

, TEHÁT

$$z_2 = 4\cdot\left(\cos\left(-\frac{\pi}{6}\right)+i\cdot\sin\left(-\frac{\pi}{6}\right)\right)$$

, ÍGY

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\cdot\left(\cos(\varphi_1-\varphi_2)+i\cdot\sin(\varphi_1-\varphi_2)\right)$$

$$= \left(\frac{3\cdot\sqrt{2}}{4}\right)\cdot\left(\cos\left(\frac{3}{4}\pi+\frac{1}{6}\pi\right)+i\cdot\sin\left(\frac{3}{4}\pi+\frac{1}{6}\pi\right)\right)$$

$$= \left(\frac{3\cdot\sqrt{2}}{4}\right)\cdot\left(\cos\left(\frac{11}{12}\pi\right)+i\cdot\sin\left(\frac{11}{12}\pi\right)\right)$$

$$\boxed{\begin{aligned} x &= 1+t \\ y &= -t \\ z &= -1+2t \end{aligned}}$$

$\rightarrow$ IRÁNYVEKTORA: $(1,-1,2) = \underline{v}_1$

$\rightarrow$ EGY PONT RAJTA: $P_1(1,0,-1)$

$$\boxed{\begin{aligned} x &= -t \\ y &= -1+t \\ z &= 1+t \end{aligned}}$$

$\rightarrow$ IRÁNY: $(-1,1,1) = \underline{v}_2$

$\rightarrow$ EGY PONT: $P_2(0,-1,1)$

$$\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 2 \\ -1 & 1 & 1 \end{vmatrix} = \underline{i}\cdot((-1)\cdot 1-2) + \underline{j}\cdot(2\cdot(-1)-1) + \underline{k}\cdot((-1)\cdot(-1)-1\cdot 1) = (-3,-3,0) = \underline{b}$$

$$\overrightarrow{P_1P_2} = (-1,-1,2) = \underline{a}$$

$\underline{a}$ VETÜLETE $\underline{b}$ EGYENESÉRE:

$$\left(\frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\right)\cdot\underline{b} = \frac{6}{18}\cdot\underline{b} = \frac{1}{3}\cdot\underline{b} = (-1,-1,0) = \underline{w}$$

TEHÁT AZ EGYENESEK TÁVOLSÁGA:

$$|\underline{w}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$$

(LÁSD SZENNELT JEGYZET 18.–19. OLDAL)

(4) $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n^2+3}{6-12n^2} = \frac{2}{-12} = -\frac{1}{6} = A$$

KELL: N, HOGY $\boxed{n \geq N} \Rightarrow \boxed{|a_n - A| < \varepsilon}$

AZAZ: $$\left| \frac{2n^2+3}{6-12n^2} + \frac{1}{6} \right| < \frac{1}{300}$$ KELL

$$\left| \frac{(2n^2+3)+(1-2n^2)}{6-12n^2} \right| < \frac{1}{300}$$

$$\boxed{\frac{4}{|6-12n^2|} < \frac{1}{300}} \qquad \boxed{1200 < |6-12n^2|}$$

KELL

$$\boxed{1200 < 12n^2 - 6} \qquad \boxed{\frac{1206}{12} < n^2}$$

$\boxed{100 + \frac{1}{2} < n^2}$ HA $\boxed{n \geq 11}$, AKKOR TELJESÜL EZ, TEHÁT $\boxed{N = 11}$