① EGYENES IRÁNYVEKTORA: $(1,2,-1)$

PONTON ÁTMENŐ SÍK EZZEL A NORMÁLVEKTORRAL:

$1\cdot(x-1)+2\cdot(y-5)+(-1)\cdot(z-1)=0$

SÍK ÉS EGYENES METSZÉSPONTJA:

$1\cdot(1+t-1)+2\cdot(2t-5)+(-1)\cdot(3-t-1)=0$

$\boxed{t=2}$ $(1+t,\ 2t,\ 3-t)=(3,4,1)$

$(3,4,1)-(1,5,1)=(2,-1,0)$

TEHÁT A TÁVOLSÁG: $d=\sqrt{2^2+(-1)^2+0^2}=\sqrt{5}$

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② a) $\lim\limits_{n\to\infty} a_n = A$, HA

$\forall \varepsilon>0 \ \exists n_0 : \forall n\geq n_0 : |a_n-A|<\varepsilon$

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b) (a_n) DIVERGENS, MERT $|a_n|\geq\sqrt{n}\to\infty$

(b_n) DIVERGENS, MERT $\lim\limits_{n\to\infty}|b_n|=1$,

DE A PÁROS INDEXŰ TAGOK 1-HEZ TARTANAK

A PÁRATLAN INDEXŰ TAGOK −1 —"—

(c_n) KONVERGENS, MERT $c_n=\dfrac{2n+1}{n^2+1}\cdot(-1)^n$

TEHÁT $|c_n|\to 0$ AHOGY $n\to\infty$

ÉS AKKOR $c_n\to 0$.

1. OLDAL

③ IMPLICIT: $\boxed{(x-2)^2+y^2=4}$

POLÁR: $(r(\varphi)\cdot\cos(\varphi)-2)^2+(r(\varphi)\cdot\sin(\varphi))^2=4$

$$r^2(\varphi)\cos^2(\varphi)-4r(\varphi)\cdot\cos(\varphi)+4+r^2(\varphi)\cdot\sin^2(\varphi)=4$$

$$r^2(\varphi)=4\cdot r(\varphi)\cdot\cos(\varphi)$$

$\boxed{r(\varphi)=4\cdot\cos(\varphi)}$ ← POLÁR

PARAMÉTERES: PL: $\boxed{\begin{array}{l} x(t)=2+2\cdot\cos(t) \\ y(t)=2\cdot\sin(t)\end{array}}$

EXPLICIT: $\boxed{y=-\sqrt{4-(x-2)^2}}$

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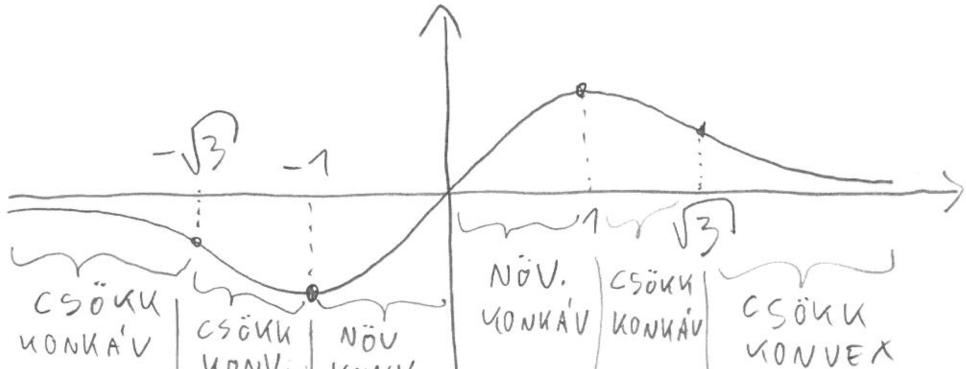
④ $f'(x)=e^{-x^2/2}+x\cdot e^{-x^2/2}\cdot(-x)=e^{-x^2/2}\cdot(1-x^2)$

$$f''(x)=e^{-x^2/2}\cdot(-x)\cdot(1-x^2)+e^{-x^2/2}\cdot(-2x)$$

$$=e^{-x^2/2}\cdot(-x)\cdot(1-x^2+2)=e^{-x^2/2}\cdot(-x)\cdot(3-x^2)$$



(5) a) $f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2} \cdot x^{-1/2}$

$$f''(x) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot x^{-3/2} \qquad f'''(x) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{3}{2}\right) \cdot x^{-5/2}$$

$$f^{(4)}(x) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{3}{2}\right) \cdot \left(-\frac{5}{2}\right) \cdot x^{-7/2}$$

$$T_3(x) = f(1) + f'(1) \cdot (x-1) + \frac{1}{2} \cdot f''(1) \cdot (x-1)^2 + \frac{1}{6} \cdot f'''(1) \cdot (x-1)^2$$

$$= 1 + \frac{1}{2} \cdot (x-1) - \frac{1}{8} \cdot (x-1)^2 + \frac{1}{16} \cdot (x-1)^3$$

b) $T_3(1.2) = 1 + \frac{1}{2} \cdot (0,2) - \frac{1}{8} \cdot (0,2)^2 + \frac{1}{16} \cdot (0,2)^3$

c) $|R_3(1,2)| \leq \max\limits_{1 \leq b \leq 1.2} \left| \frac{f^{(4)}(b)}{24} \right| \cdot (0.2)^4 =$

$$= \max_{1 \leq b \leq 1.2} \frac{15}{16} \cdot \frac{1}{24} \cdot b^{-7/2} \cdot (0.2)^4 = \frac{15}{16} \cdot \frac{1}{24} \cdot (0,2)^4$$

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(6) $\int (\ln(x))^{-1/2} \cdot \ln'(x)\, dx = \frac{1}{-\frac{1}{2}+1} \cdot \ln(x)^{-\frac{1}{2}+1} + C$

$$= 2 \cdot \sqrt{\ln(x)} + C$$

$$\int \frac{1}{2} \cdot \frac{(x^2-2x-1)'}{x^2-2x-1}\, dx = \frac{1}{2} \ln(|x^2-2x-1|) + C$$

$$\int \frac{1}{(x+2)^2+2^2}\, dx = \frac{1}{2} \arctan\left(\frac{x+2}{2}\right) + C$$

3. OLDAL

⑦ $\int x^3 \cdot \sin(2x^2)\, dx =$

$u = 2x^2$ | $x^2 = \frac{1}{2}u$

$\frac{du}{dx} = 4x$

$x \cdot dx = \frac{1}{4}du$

$\int x^2 \cdot \sin(2x^2) \cdot x \cdot dx =$

$\int \frac{1}{2}u \cdot \sin(u) \cdot \frac{1}{4}\, du = \frac{1}{8} \cdot \int u \cdot \sin(u)\, du =$

PARCIÁLIS INT

$\frac{1}{8} \cdot \left(u \cdot (-\cos(u)) - \int 1 \cdot (-\cos(u))\, du \right) =$

$\frac{1}{8} \cdot \left(\sin(u) - u \cdot \cos(u) \right) + C =$

$= \frac{1}{8} \cdot \left(\sin(2x^2) - 2x^2 \cdot \cos(2x^2) \right) + C$

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⑧ $\int_0^1 \sqrt{1 + (f'(x))^2}\, dx = \int_0^1 \sqrt{1 + x^2}\, dx =$ $\boxed{x = \mathrm{sh}(u)}$

$= \int_0^{\mathrm{arsh}(1)} \sqrt{1 + \mathrm{sh}^2(u)} \cdot \mathrm{ch}(u)\, du = \int_0^{a(1)} \mathrm{ch}^2(u)\, du =$

$= \int_0^{a(1)} \frac{\mathrm{ch}(2u) + 1}{2}\, du = \frac{1}{2}a(1) + \frac{1}{2}\left[\frac{1}{2}\mathrm{sh}(2u) \right]_0^{a(1)} =$

$= \frac{1}{2}\mathrm{arsh}(1) + \frac{1}{4} \cdot 2 \cdot \mathrm{sh}(a(1)) \cdot \mathrm{ch}(a(1)) =$

$= \frac{1}{2}\mathrm{arsh}(1) + \frac{1}{2} \cdot 1 \cdot \sqrt{1 + 1^2} \approx 1.1478$

9)
$$T=\int_0^\infty e^{-x/2}\,dx=\lim_{b\to\infty}\int_0^b e^{-x/2}\,dx=$$

$$=\lim_{b\to\infty}\left[-2\cdot e^{-x/2}\right]_0^b=2-\lim_{b\to\infty}2\cdot e^{-b/2}=2$$

$$M_x=\frac{1}{2}\int_0^\infty \left(e^{-x/2}\right)^2 dx=\frac{1}{2}\int_0^\infty e^{-x}\,dx=\ldots=\frac{1}{2}$$

TEHÁT $S_y=\dfrac{M_x}{T}=\dfrac{1}{4}$

$$M_y=\int_0^\infty x\cdot e^{-x/2}\,dx=\lim_{b\to\infty}\int_0^b x\cdot e^{-x/2}\,dx\overset{\text{PARCIÁLIS}}{=}$$

$$=\underbrace{\lim_{b\to\infty}\left[x\cdot(-2)\cdot e^{-x/2}\right]_0^b}_{=\,0-0=0}-\lim_{b\to\infty}\int_0^b 1\cdot(-2)\cdot e^{-x/2}\,dx=$$

$$=2\cdot\lim_{b\to\infty}\left[(-2)\cdot e^{-x/2}\right]_0^b=2\cdot\left(0-(-2)\right)=4$$

TEHÁT $S_x=\dfrac{M_y}{T}=\dfrac{4}{2}=2$