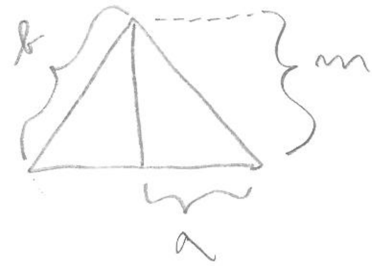
C CSOPORT

(1) a) LÁSD 109. OLDAL

b) LÁSD 111. OLDAL ALJA, 112. OLDAL

(2) 

$b = \sqrt{a^2+m^2}$

$K = 2a + 2b = 2 \cdot \left(a + \sqrt{a^2+m^2}\right) = 1$

$T = a \cdot m$ $\quad \boxed{0 < a < \frac{1}{4}}$

$a + \sqrt{a^2+m^2} = \frac{1}{2} \iff \sqrt{a^2+m^2} = \frac{1}{2} - a \iff$

$a^2 + m^2 = \left(\frac{1}{2} - a\right)^2 \iff m^2 = \left(\frac{1}{2}-a\right)^2 - a^2 = \frac{1}{4} - a$

MELY a ESETÉN MAXIMÁLIS T? $\iff$

-||- -||- -||- -||- T^2?

$T^2 = a^2 \cdot m^2 = a^2 \cdot \left(\frac{1}{4} - a\right) = \frac{1}{4}a^2 - a^3 = f(a)$

$\boxed{f'(a) = 0} \iff \frac{1}{2}a - 3a^2 = 0 \iff a \cdot \left(\frac{1}{2} - 3a\right) = 0$

$\boxed{\frac{1}{2} - 3a = 0} \implies \boxed{a = \frac{1}{6}} \implies \boxed{m = \sqrt{\frac{1}{4} - \frac{1}{6}} = \sqrt{\frac{1}{12}}}$

$\boxed{a \neq 0}$

3) a) $f(x) = x^{1/3}$ $\quad f'(x) = \frac{1}{3} \cdot x^{-2/3}$

$$f''(x) = \frac{1}{3} \cdot \left(-\frac{2}{3}\right) \cdot x^{-5/3} = -\frac{2}{9} \cdot x^{-5/3}$$

$f(1) = 1 \quad f'(1) = \frac{1}{3} \quad f''(1) = -\frac{2}{9}$

$$T_2(x) = 1 + \frac{1}{3} \cdot (x-1) + \frac{1}{2} \cdot \left(-\frac{2}{9}\right) \cdot (x-1)^2$$

b) KÖZELÍTÉS: $T_2(0.5) = 1 + \frac{1}{3} \cdot (0.5-1) - \frac{1}{9} \cdot (0.5-1)^2$

<u>TAYLOR</u>-<u>TÉTEL</u>: VAN OLYAN $\frac{1}{2} \le b \le 1$, HOGY

$$R_2(0.5) = \frac{f'''(b)}{3!} \cdot (0.5-1)^2 = ✪$$

$$f'''(x) = -\frac{2}{9} \cdot \left(-\frac{5}{3}\right) \cdot x^{-8/3} = \frac{10}{27} \cdot x^{-8/3} \leftarrow \boxed{\text{CSÖKKENŐ FÜGGVÉNY}}$$

$$\max_{\frac{1}{2} \le b \le 1} f'''(b) = f'''\left(\frac{1}{2}\right) = \frac{10}{27} \cdot (0.5)^{-8/3}$$

<u>HIBA</u> <u>BECSLÉS</u>: $|R_2(0.5)| = |✪| \le$

$$\le \frac{1}{3!} \cdot \frac{10}{27} \cdot (0.5)^{-8/3} \cdot (0.5)^3 =$$

$$= \frac{1}{6} \cdot \frac{10}{27} \cdot \sqrt[3]{0.5}$$

④ a) $\int \underbrace{x}_{g} \cdot \underbrace{\sin(x)}_{f'} \, dx = \underbrace{-\cos(x)}_{f} \cdot \underbrace{x}_{g} - \int \underbrace{(-\cos(x))}_{f} \cdot \underbrace{1}_{g'} \, dx =$

$= -\cos(x) \cdot x + \sin(x) + C$

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b) $\int \sin(\sqrt{x}) \, dx =$ $\boxed{u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2} \frac{1}{\sqrt{x}} = \frac{1}{2} \cdot \frac{1}{u} \;\Big|\; dx = 2u\,du}$

$= \int \sin(u) \cdot 2u \, du = 2 \cdot (-\cos(u) \cdot u + \sin(u)) + C$

$= 2 \cdot (-\cos(\sqrt{x}) \cdot \sqrt{x} + \sin(\sqrt{x})) + C$

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⑤ $\frac{2x-1}{(x-1)^2 \cdot x} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x}$

$2x-1 = A \cdot (x-1) \cdot x + B \cdot x + C \cdot (x-1)^2$

$\boxed{x=1} \Rightarrow 1 = A \cdot 0 + B + C \cdot 0 \Rightarrow \boxed{B=1}$

$\boxed{x=0} \Rightarrow -1 = A \cdot 0 + B \cdot 0 + C \cdot (-1)^2 \Rightarrow \boxed{C=-1}$

$2x-1 = A \cdot (x^2 - x) + x - (x-1)^2 \Rightarrow$

$A \cdot (x^2 - x) = 2x - 1 + (x-1)^2 - x = x - 1 + x^2 - 2x + 1 = x^2 - x$

$\boxed{A=1}$

$\frac{2x-1}{(x-1)^2 \cdot x} = \frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{x}$

3. OLDAL

$$\int \frac{2x-1}{(x-1)^2 \cdot x}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx - \int \frac{1}{x}\,dx =$$

$$= \ln|x-1| - \frac{1}{x-1} - \ln|x| + C$$

b) $x^3 - 2x^2 + x = x \cdot (x^2 - 2x + 1) = x \cdot (x-1)^2$

$$\int \frac{x^3 - 2x^2 + 3x - 1}{x^3 - 2x^2 + x}\,dx = \int \left(1 + \frac{2x-1}{x^3 - 2x^2 + x}\right) dx =$$

$$= x + \ln|x-1| - \frac{1}{x-1} - \ln|x| + C$$

4.OLDAC