

# A CSOPORT:

$$\textcircled{1} \text{ a) } T_n(x) = f(a) + f'(a) \cdot (x-a) + \frac{f''(a)}{2!} \cdot (x-a)^2 + \dots \\ + \dots + \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$

$$R_n(x) = f(x) - T_n(x)$$

TAYLOR-TÉTEL: LÁSD SZ KENNECT  
ZEGYZET 105. OLDAL ALTA.

$\textcircled{1}$  b) (LÁSD 103. OLDAL)

$$T_1(x) = f(5) + f'(5) \cdot (x-5) = 2 + (-1) \cdot (x-5) \\ = 7 - x$$

$$R_1(x) = f(x) - T_1(x) = x^2 - 11x + 32 - (7 - x) = \\ = x^2 - 10x + 25 = (x-5)^2$$

$$|f(7) - T_1(7)| = |R_1(7)| \stackrel{\text{TAYLOR-TÉTEL}}{\leq} \max_{5 \leq b \leq 7} \left| \frac{f''(b)}{2!} \cdot (x-5)^2 \right| =$$

$$= \max_{5 \leq b \leq 7} \left| \frac{2}{2!} \cdot (x-5)^2 \right| = \max_{5 \leq b \leq 7} (x-5)^2 = (7-5)^2 = 4$$

$$R_1(7) \stackrel{\star}{=} (7-5)^2 = 4 \leftarrow \text{TÉNYLEGES ÉRTÉK}$$

BECSLÉS

1. OLDAL

$$(2) \quad 6x_0^2 + 3x_0 \cdot y_0 + 2y_0^2 + 17y_0 - 6 = 0 \quad (?)$$

$$6 \cdot (-1)^2 + 3 \cdot (-1) \cdot 0 - 2 \cdot 0^2 + 17 \cdot 0 - 6 = 0 \quad (?)$$

$$6 - 6 = 0 \quad \checkmark \quad \text{TÉNYLEG ILLESZKEDIK!}$$

$$6x^2 + 3x \cdot y(x) + 2y^2(x) + 17y(x) - 6 = 0$$

DERIVÁLCZUK MINDKÉT OLDALT:

$$12x + 3y(x) + 3x \cdot y'(x) + 4y(x) \cdot y'(x) + 17 \cdot y'(x) = 0$$

BEHELYETTÉSÍTÜNK  $x_0 = -1$ -ET ÉS  $y(x_0) = 0$ -T:

$$12 \cdot (-1) + 3 \cdot 0 + 3 \cdot (-1) \cdot y'(x_0) + 4 \cdot 0 \cdot y'(x_0) + 17 \cdot y'(x_0) = 0$$

$$-12 - 3 \cdot y'(x_0) + 17 \cdot y'(x_0) = 0$$

$$14y'(x_0) = 12 \quad \text{AZAZ} \quad y'(x_0) = \frac{6}{7}$$

ÉRINTŐ  
EGYENES  
MEREDKSÉGE

EGYENES EGYENLETE:  $y - y_0 = m \cdot (x - x_0)$  AZAZ

$$y - 0 = \frac{6}{7} \cdot (x - (-1)) \quad \text{AZAZ} \quad y = \frac{6}{7} \cdot (x + 1)$$

$$3) f(x) = \sqrt{2x - x^2}$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{2x - x^2}} \cdot (2 - 2x) = \frac{1-x}{\sqrt{2x - x^2}}$$

$$f''(x) = \frac{(1-x)' \cdot \sqrt{2x - x^2} - (1-x) \cdot \frac{1}{2} \cdot (2x - x^2)^{-1/2} \cdot (2 - 2x)}{\sqrt{2x - x^2}^2} =$$

$$= \frac{-\sqrt{2x - x^2} - (1-x)^2 \cdot (2x - x^2)^{-1/2}}{2x - x^2}$$

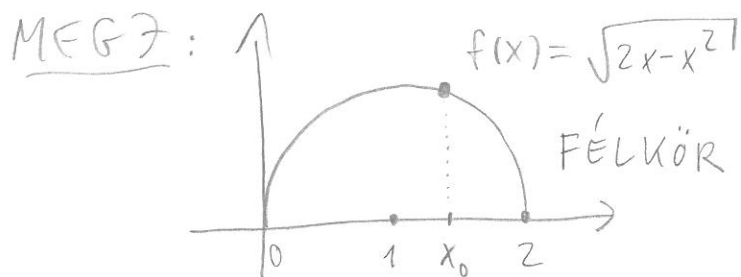
$$f'(x_0) = \frac{1 - 3/2}{\sqrt{2 \cdot 3/2 - (3/2)^2}} = \frac{-1/2}{\sqrt{3 - 9/4}} = \frac{-1/2}{\sqrt{3/4}} = \frac{-1}{\sqrt{3}}$$

$$f''(x_0) = \frac{-\sqrt{3 - (3/2)^2} - (1 - 3/2)^2 \cdot (3 - 9/4)^{-1/2}}{3 - 9/4} =$$

$$= \frac{-\sqrt{3/4} - 1/4 \cdot (3/4)^{-1/2}}{3/4} = \frac{-3/4 - 1/4}{\sqrt{3/4} \cdot (3/4)} = \frac{-1}{(3/4)^{3/2}} = -\left(\frac{4}{3}\right)^{3/2}$$

$$G(x_0) = \frac{-\left(\frac{4}{3}\right)^{3/2}}{\left(1 + \frac{1}{3}\right)^{3/2}} = -1$$

A KERESETT GÖRBÜLET



3. OLDAL

$$\textcircled{4} \text{ a) } \int \frac{x^3 - x + 1}{\sqrt{x}} dx = \int (x^{5/2} - x^{1/2} + x^{-1/2}) dx$$

$$= \frac{1}{\frac{5}{2}+1} \cdot x^{\frac{5}{2}+1} - \frac{1}{\frac{1}{2}+1} \cdot x^{\frac{1}{2}+1} + \frac{1}{-\frac{1}{2}+1} \cdot x^{-\frac{1}{2}+1} + C =$$

$$= \frac{2}{7} \cdot x^{7/2} - \frac{2}{3} \cdot x^{3/2} + 2 \cdot \sqrt{x} + C$$


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$$\textcircled{4} \text{ b) } \int x^2 \cdot \operatorname{sh}\left(\frac{x}{2}\right) dx = \text{PARCIA'LIS INT}$$

$$= x^2 \cdot 2 \cdot \operatorname{ch}\left(\frac{x}{2}\right) - \int 2x \cdot 2 \operatorname{ch}\left(\frac{x}{2}\right) dx =$$

$$= x^2 \cdot 2 \cdot \operatorname{ch}\left(\frac{x}{2}\right) - 4 \cdot \int x \cdot \operatorname{ch}\left(\frac{x}{2}\right) dx = \text{PARCIA'LIS INT}$$

$$= 2x^2 \cdot \operatorname{ch}\left(\frac{x}{2}\right) - 4 \cdot \left( x \cdot 2 \cdot \operatorname{sh}\left(\frac{x}{2}\right) - \int 2 \cdot \operatorname{sh}\left(\frac{x}{2}\right) dx \right)$$

$$= 2x^2 \cdot \operatorname{ch}\left(\frac{x}{2}\right) - 8x \cdot \operatorname{sh}\left(\frac{x}{2}\right) + 16 \operatorname{ch}\left(\frac{x}{2}\right) + C$$

$$= (2x^2 + 16) \cdot \operatorname{ch}\left(\frac{x}{2}\right) - 8x \cdot \operatorname{sh}\left(\frac{x}{2}\right) + C$$