A CSOPORT

① a) LÁSD 103. OLDAL, 105. OLDAL

b) 105. OLDAL

c) $f(x)$ EGY MÁSODFOKÚ POLINOM, TEHÁT $T_2(x) = f(x)$, TEHÁT $R_2(x) \equiv 0$

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② $$G(x) = \frac{f''(x)}{\left(1+(f'(x))^2\right)^{3/2}} = \frac{e^{-x}}{\left(1+e^{-2x}\right)^{3/2}}$$

$$G'(x) = \frac{-e^{-x}\cdot\left(1+e^{-2x}\right)^{3/2} - e^{-x}\cdot\frac{3}{2}\cdot\left(1+e^{-2x}\right)^{1/2}\cdot e^{-2x}\cdot(-2)}{\left(1+e^{-2x}\right)^3}$$

$$= \frac{e^{-x}\cdot\left(1+e^{-2x}\right)^{1/2}\cdot\left(3\cdot e^{-2x} - 1 - e^{-2x}\right)}{\left(1+e^{-2x}\right)^3} = \frac{e^{-x}}{\left(1+e^{-2x}\right)^{5/2}}\cdot\left(2e^{-2x}-1\right)$$

$$G'(x) = 0 \iff 2e^{-2x} - 1 = 0 \iff e^{-2x} = \frac{1}{2} \Rightarrow$$

$\Rightarrow \boxed{x_0 = -\frac{1}{2}\ln\left(\frac{1}{2}\right)}$ ÉS VALÓBAN MAXIMUMHELY,

MIVEL $x < x_0 \Rightarrow G'(x) > 0$, $G\nearrow$
$x > x_0 \Rightarrow G'(x) < 0$, $G\downarrow$ $\Bigg\}$ $\boxed{\text{LÁSD 113. OLDAL}}$

③ $f(x) = \sin\left(\frac{x}{2}\right)$ $f'(x) = \cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$f''(x) = -\sin\left(\frac{x}{2}\right)\cdot\frac{1}{4}$ $f'''(x) = -\cos\left(\frac{x}{2}\right)\cdot\frac{1}{8}$

$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $f'\left(\frac{\pi}{2}\right) = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$

$f''\left(\frac{\pi}{2}\right) = -\frac{1}{4}\cdot\frac{1}{\sqrt{2}}$ $f'''\left(\frac{\pi}{2}\right) = -\frac{1}{8}\cdot\frac{1}{\sqrt{2}}$

$$T_3(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right) + \frac{1}{2}f''\left(\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right)^2$$
$$+\frac{1}{6}\cdot f'''\left(\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right)^3 =$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\left(x-\frac{\pi}{2}\right) - \frac{1}{8}\cdot\frac{1}{\sqrt{2}}\cdot\left(x-\frac{\pi}{2}\right)^2 - \frac{1}{48}\cdot\frac{1}{\sqrt{2}}\cdot\left(x-\frac{\pi}{2}\right)^3$$

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④ a) $\int \underbrace{x}_{g}\cdot\underbrace{ch(x)}_{f'}\,dx = \underbrace{sh(x)}_{f}\cdot\underbrace{x}_{g} - \int \underbrace{sh(x)}_{f}\cdot\underbrace{1}_{g'}\,dx =$

$= sh(x)\cdot x - ch(x)$

$u = x^2$ $\frac{du}{dx} = 2x$	$\frac{1}{2}du = x\,dx$

b) $\int x^3\cdot ch(x^2)\,dx =$

$= \int x^2\cdot ch(x^2)\,x\,dx = \frac{1}{2}\int u\cdot ch(u)\,du = \boxed{\text{LAŚSD a)}}$

$= \frac{1}{2}\cdot\left(sh(u)\cdot u - ch(u)\right) + C = \frac{1}{2}\cdot\left(sh(x^2)\cdot x^2 - ch(x^2)\right) + C$

2. OLDAL

5 a) $\int \frac{1}{(x-2)^2+1}\,dx = \arctan(x-2) + C$

b) $\frac{1}{2}\int \frac{(x^2-4x+5)'}{(x^2-4x+5)}\,dx = \frac{1}{2}\ln(x^2-4x+5) + C$

c) $\int \frac{4x-5}{x^2-4x+5}\,dx = \int \frac{2\cdot(2x-4)}{x^2-4x+5}\,dx +$

$+ \int \frac{3}{x^2-4x+5}\,dx = 2\cdot\ln(x^2-4x+5) + 3\cdot\arctan(x-2) + C$

d) $\int \frac{x^2}{x^2-4x+5}\,dx = \int\left(\frac{x^2-4x+5}{x^2-4x+5} + \frac{4x-5}{x^2-4x+5}\right)dx =$

$= \int 1\,dx + \int \frac{4x-5}{x^2-4x+5}\,dx =$

$x + 2\cdot\ln(x^2-4x+5) + 3\cdot\arctan(x-2) + C$

3.0CDAC