1) a)

$\underline{i}$	$\underline{j}$	$\underline{k}$	$\underline{i}$	$\underline{j}$	$\underline{k}$
x_1	y_1	z_1	x_1	y_1	z_1
x_2	y_2	z_2	x_2	y_2	z_2

$$\underline{v}_1 \times \underline{v}_2 = \underline{i}\cdot(y_1\cdot z_2 - z_1\cdot y_2) + \underline{j}\cdot(z_1\cdot x_2 - x_1\cdot z_2) +$$
$$+ \underline{k}\cdot(x_1\cdot y_2 - y_1\cdot x_2)$$

TEHÁT $x_3 = y_1\cdot z_2 - z_1\cdot y_2$

$y_3 = z_1\cdot x_2 - x_1\cdot z_2$

$z_3 = x_1\cdot y_2 - y_1\cdot x_2$

---

b) $(\underline{v}_1 \times \underline{v}_2)\cdot \underline{v}_1 = x_3\cdot x_1 + y_3\cdot y_1 + z_3\cdot z_1 =$

$= y_1\cdot z_2\cdot x_1 - z_1\cdot y_2\cdot x_1 + z_1\cdot x_2\cdot y_1 - x_1\cdot z_2\cdot y_1 +$

$+ x_1\cdot y_2\cdot z_1 - y_1\cdot x_2\cdot z_1 = 0 + 0 + 0 = 0$

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c) ÁLTALÁBAN : $|\underline{v}_1 \times \underline{v}_2| = |\underline{v}_1|\cdot|\underline{v}_2|\cdot \sin(\varphi)$

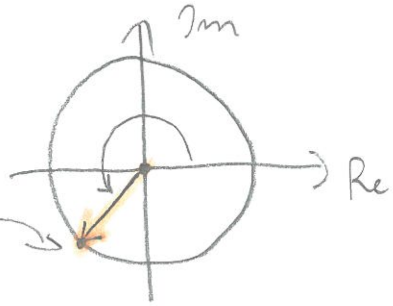
(LÁSD JEGYZET 10. OLDALA)

ITT: $|\underline{v}_1| = 2$, $|\underline{v}_2| = 3$, $\varphi = \frac{\pi}{2} \Rightarrow \sin(\varphi) = 1$

TEHÁT $|\underline{v}_1 \times \underline{v}_2| = 2\cdot 3\cdot 1 = 6$

② a) $|z_1| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

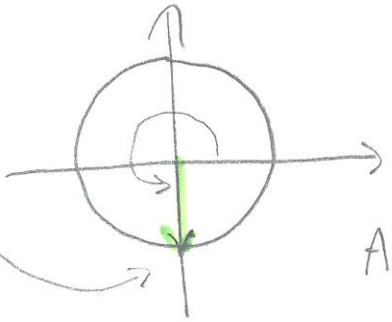
$$\boxed{\frac{z_1}{|z_1|} = \frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}} :$$



TEHÁT $Arg(z_1) = \pi + \frac{\pi}{4} = \frac{5}{4}\pi$

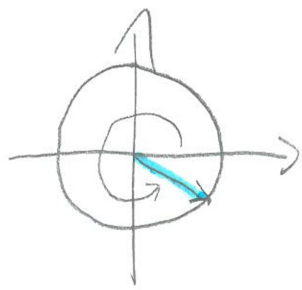
$$\boxed{z_2 = \frac{1}{i} = -i}$$

$|z_2| = 1$



$Arg(z_2) = \frac{3}{2}\pi$

$|z_3| = \sqrt{\sqrt{12}^2 + (-2)^2} = 4$



$$\boxed{\frac{z_3}{|z_3|} = \frac{\sqrt{12}}{4} - \frac{2}{4}i = \frac{\sqrt{3}}{2} - \frac{1}{2}i}$$

$Arg(z_3) = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$z_1 = \sqrt{2} \cdot \left(\cos\left(\frac{5}{4}\pi\right) + i \cdot \sin\left(\frac{5}{4}\pi\right)\right)$

$z_2 = 1 \cdot \left(\cos\left(\frac{3}{2}\pi\right) + i \cdot \sin\left(\frac{3}{2}\pi\right)\right)$

$z_3 = 4 \cdot \left(\cos\left(\frac{11}{6}\pi\right) + i \cdot \sin\left(\frac{11}{6}\pi\right)\right)$

② b) $|z_4| = \dfrac{|z_1|}{|z_2|\cdot|z_3|} = \dfrac{\sqrt{2}}{1\cdot 4} = \dfrac{\sqrt{2}}{4}$

$$\mathrm{Arg}(z_4) = \mathrm{Arg}(z_1) - \mathrm{Arg}(z_2) - \mathrm{Arg}(z_3)$$

$$= \frac{5}{4}\pi - \frac{3}{2}\pi - \frac{11}{6}\pi = \frac{15-18-22}{12}\pi$$

$$= \frac{-25}{12}\pi = -2\pi - \frac{1}{12}\pi = 2\pi - \frac{1}{12}\pi = \frac{23}{12}\pi$$

SZÖGEKRŐL VAN SZÓ

$$z_4 = \frac{\sqrt{2}}{4}\cdot\left(\cos\left(\frac{23}{12}\pi\right) + i\cdot\sin\left(\frac{23}{12}\pi\right)\right)$$

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③ $\vec{PQ} = (-1,-1,-1)$ $\quad \vec{PR} = (0,0,-1)$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -1 & -1 \\ 0 & 0 & -1 \end{vmatrix} = \underline{i}\cdot(1-0) + \underline{j}\cdot(0-1) + \underline{k}\cdot(0-0)$$

$$= (1,-1,0)$$

a) $T_\Delta = \frac{1}{2}\cdot|\vec{PQ}\times\vec{PR}| = \frac{1}{2}\cdot\sqrt{1^2+(-1)^2+0^2} = \frac{\sqrt{2}}{2}$

b) SÍK NORMÁLVEKTORA: $\vec{PQ}\times\vec{PR} = (1,-1,0)$

SÍK EGY PONTJA: $P = (3,-1,2)$

SÍK EGYENLETE: $1\cdot(x-3) + (-1)\cdot(y-(-1)) + 0\cdot(z-2) = 0$

AZAZ: $x-3-y-1=0$

AZAZ: $\boxed{x-y=4}$

3. OLDAL

4) a) KERESSÜK MEG A SZÁMLÁLÓ ÉS A NEVEZŐ DOMINÁNS TAGJÁT!

$\boxed{\frac{7}{3} = 2.\dot{3}}$ $\boxed{\frac{5}{2} = 2.5}$ $\boxed{\frac{12}{5} = 2.4}$ TEHÁT

$n^{5/2}$ A DOMINÁNS TAG A SZÁMLÁLÓBAN ÉS A NEVEZŐBEN IS.

OSSZUK LE A SZÁMLÁLÓT ÉS A NEVEZŐT IS A DOMINÁNS TAGGAL

$$\lim_{n\to\infty} \frac{4\cdot n^{7/4} - 3\cdot n^{5/2}}{2\cdot n^{5/2} + 5\cdot n^{12/5} + 2\cdot n^{7/3}} =$$

$$\lim_{n\to\infty} \frac{4\cdot n^{7/4-5/2} - 3}{2 + 5\cdot n^{12/5-5/2} + 2\cdot n^{7/3-5/2}} = \frac{4\cdot 0 - 3}{2+5\cdot 0+2\cdot 0} = \frac{-3}{2}$$

---

b) $a-b = \frac{a^2-b}{a+b}$, TEHÁT

$$\sqrt{n^2+3n-1} - \sqrt{n^2+2n+2} = \frac{(n^2+3n-1)-(n^2+2n+2)}{\sqrt{n^2+3n-1}+\sqrt{n^2+2n+2}}$$

$$= \frac{n-3}{\sqrt{n^2+3n-1}+\sqrt{n^2+2n+2}} = \frac{1-3/n}{\sqrt{1+3/n-1/n^2}+\sqrt{1+\frac{2}{n}+\frac{2}{n^2}}}$$

ÍGY $\lim_{n\to\infty}\left(\sqrt{n^2+3n-1} - \sqrt{n^2+2n+2}\right) =$

$$\lim_{n\to\infty} \frac{1-3/n}{\sqrt{1+3/n-1/n^2}+\sqrt{1+\frac{2}{n}+\frac{2}{n^2}}} = \frac{1-0}{\sqrt{1+0+0}+\sqrt{1+0+0}} = \frac{1}{2}$$