PL: $\boxed{f(x)=\frac{3}{2+x}}$ $\boxed{f^{-1}(x)=?}$ KI KELL FEJEZNI X-ET Y FÜGGVÉNYEKÉNT

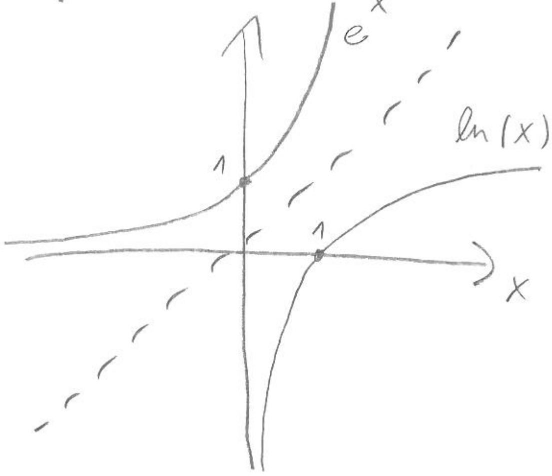
$$y=\frac{3}{2+x} \iff y\cdot(2+x)=3 \iff 2y+xy=3 \iff$$

$$\iff xy=3-2y \iff x=\frac{3-2y}{y}=\frac{3}{y}-2$$

TEHÁT $f^{-1}(y)=\frac{3}{y}-2$, AZAZ $\boxed{f^{-1}(x)=\frac{3}{x}-2}$

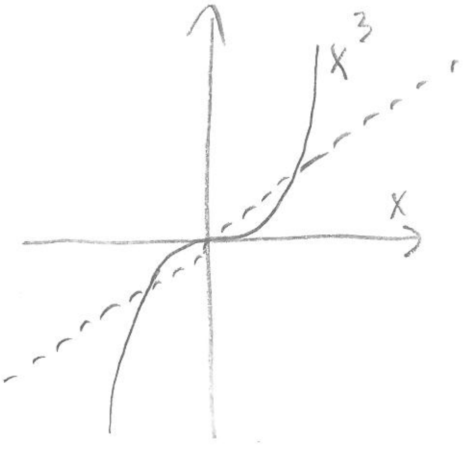
PL: $\boxed{f(x)=e^x}$, $y=e^x$, $\ln(y)=x$

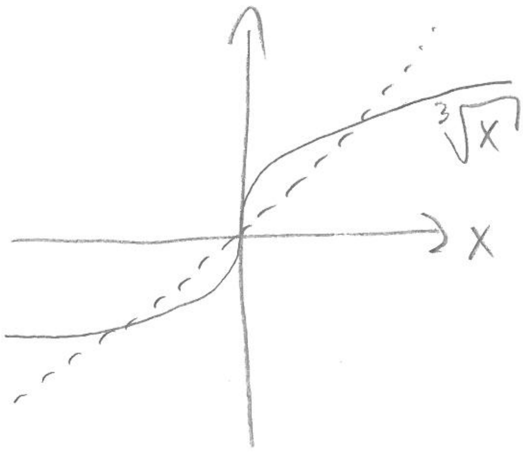
$f^{-1}(y)=\ln(y)$ $\boxed{f^{-1}(x)=\ln(x)}$



PL: $f(x)=x^3$ $\boxed{f^{-1}(x)=?}$

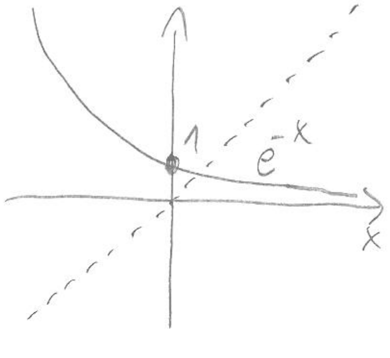
$$y=x^3 \iff x=\sqrt[3]{y} \Rightarrow f^{-1}(y)=\sqrt[3]{y} \Rightarrow \boxed{f^{-1}(x)=\sqrt[3]{x}}$$

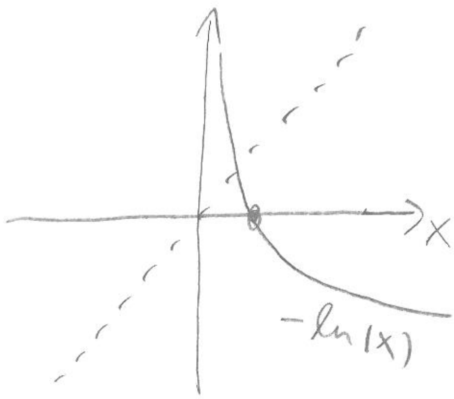




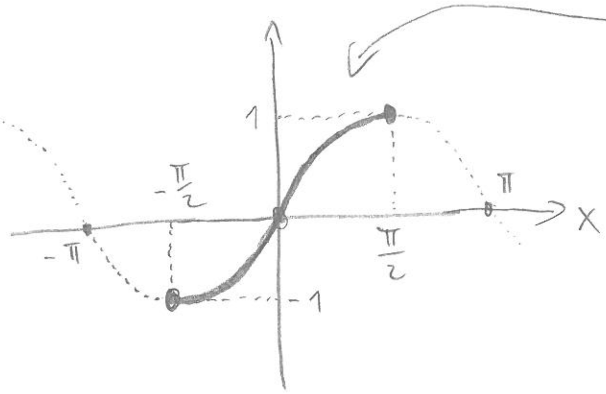
PL: $\boxed{f(x)=e^{-x}}$

$\boxed{f^{-1}(x)=-\ln(x)}$





ARKUSZFÜGGVÉNYEK = TRIG. FÜGGVÉNYEK INVERZEI



$f(x)=\sin(x)$

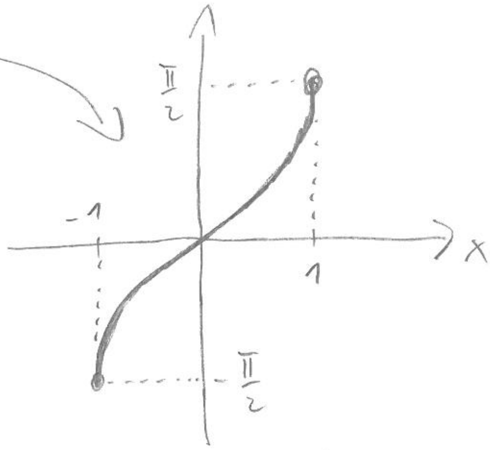
HA $\mathcal{D}_f=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, AKKOR

KÖLCS. EGYÉRTELMŰ ÉS

$\mathcal{R}_f=[-1,1]$

$f^{-1}(x)=\sin^{-1}(x)=\arcsin(x)$

$\mathcal{D}_{f^{-1}}=[-1,1]$ $\quad\mathcal{R}_{f^{-1}}=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

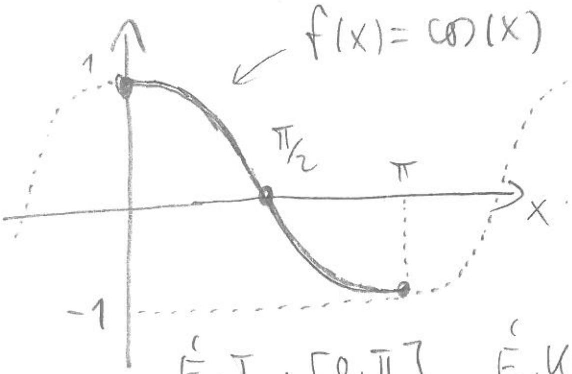


$\sin(0)=0 \Rightarrow \arcsin(0)=0$

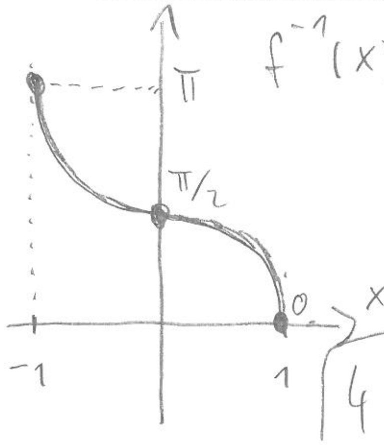
$\sin\left(-\frac{\pi}{2}\right)=-1 \Rightarrow \arcsin(-1)=-\frac{\pi}{2}$

$\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2} \Rightarrow \arcsin\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$

$f(x)=\cos(x)$



É.T.: $[0,\pi]$, É.K.: $[-1,1]$



$f^{-1}(x)=\cos^{-1}(x)$

$=\arccos(x)$

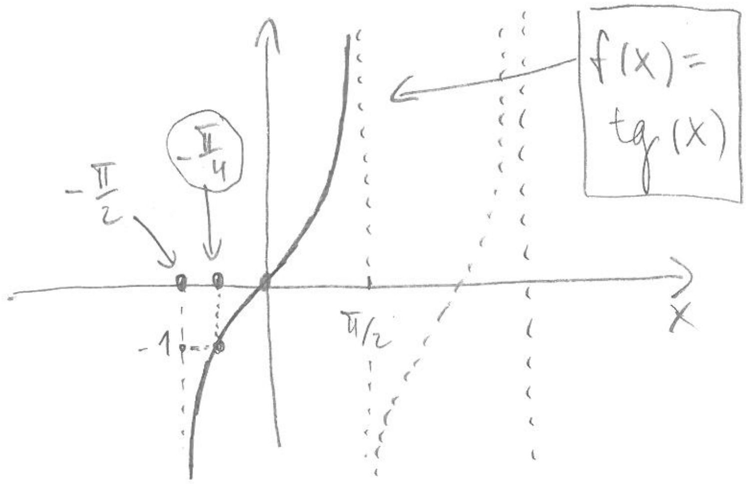
É.T.: $[-1,1]$

É.K.: $[0,\pi]$

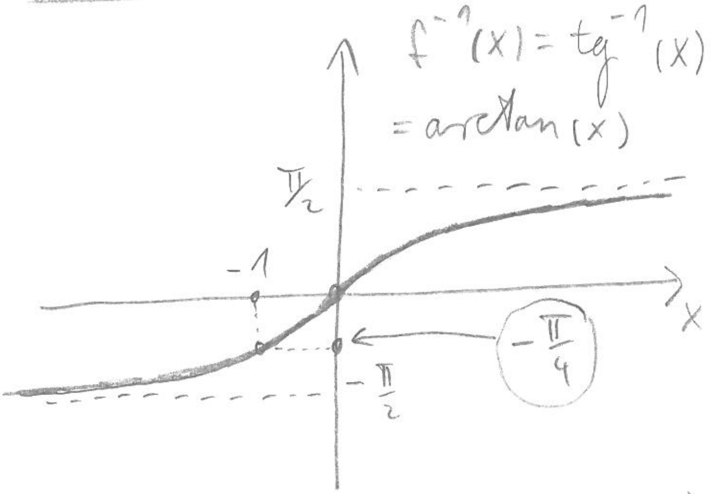
41. OLDAL

$$\cos(0)=1 \Rightarrow \arccos(1)=0 \quad \Big\| \quad \cos\left(\frac{\pi}{2}\right)=0 \Rightarrow \arccos(0)=\frac{\pi}{2}$$

$$\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \Rightarrow \arccos\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$$



$$\mathcal{D}_f=(-\pi/2, \pi/2) \quad \mathcal{R}_f=(-\infty, \infty)$$

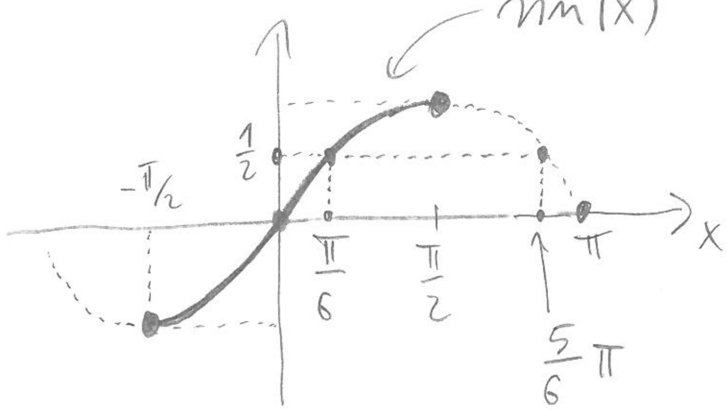


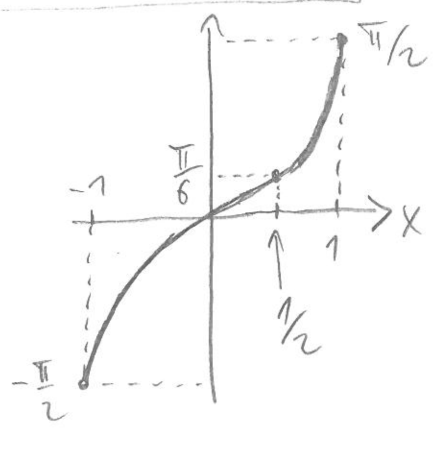
$$\mathcal{D}_{f^{-1}}=(-\infty, \infty) \quad \mathcal{R}_{f^{-1}}=\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\tan\left(-\frac{\pi}{4}\right)=\frac{\sin(-\pi/4)}{\cos(-\pi/4)}=\frac{-1/\sqrt{2}}{1/\sqrt{2}}=-1 \Rightarrow \boxed{\arctan(-1)=-\frac{\pi}{4}}$$

BEUGRATÓS: $\sin\left(\frac{5}{6}\pi\right)=\frac{1}{2}$, DE ETTŐL MÉG NEM IGAZ, HOGY $\arcsin\left(\frac{1}{2}\right)=\frac{5}{6}\pi$, HANEM

$$\boxed{\arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}}$$



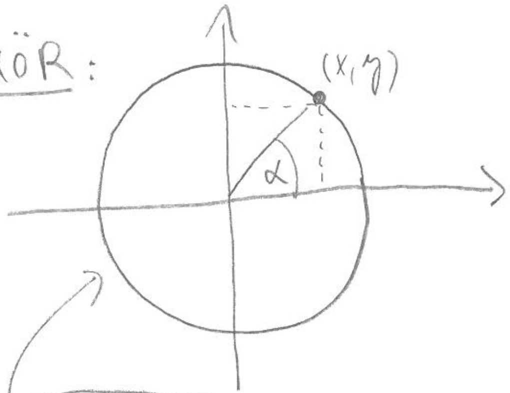


HIPERBOLIKUS FÜGGVÉNYEK:

DEF: $\boxed{ch(x) = \dfrac{e^{x} + e^{-x}}{2}}$ $\boxed{sh(x) = \dfrac{e^{x} - e^{-x}}{2}}$

MIÉRT ÉRDEKES?

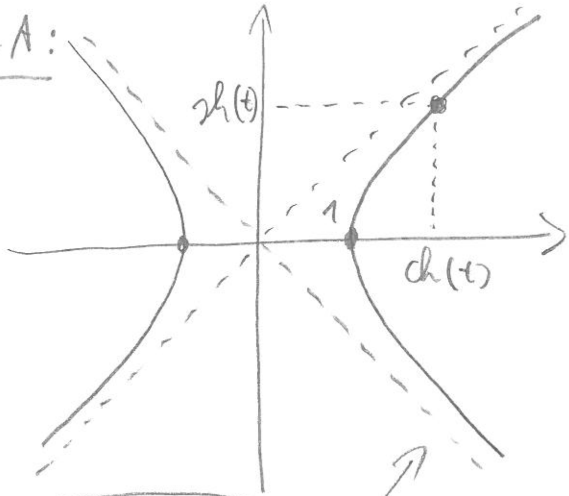
KÖR:



$\boxed{x^2 + y^2 = 1}$

$\boxed{\sin^2(\alpha) + \cos^2(\alpha) = 1}$

HIPERBOLA:



$\boxed{x^2 - y^2 = 1}$

$\boxed{ch^2(t) - sh^2(t) = 1}$

$$\left(\frac{e^{t} + e^{-t}}{2}\right)^2 - \left(\frac{e^{t} - e^{-t}}{2}\right)^2 =$$

$$= \frac{(e^{2t} + 2 + e^{-2t})}{4} - \frac{(e^{2t} - 2 + e^{-2t})}{4} = \frac{2+2}{4} = 1 \ \checkmark$$

TOVÁBBI AZONOSSÁGOK:

$\boxed{sh(2x) = 2\, sh(x)\, ch(x)}$

$\sin(2x) = 2 \sin(x) \cos(x)$

$\boxed{ch(2x) = ch^2(x) + sh^2(x)}$

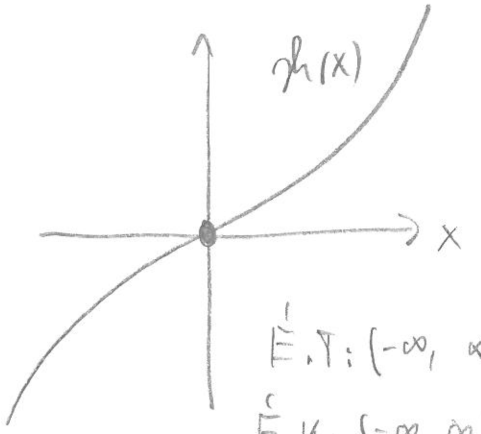
$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\boxed{ch^2(x) = \dfrac{1 + ch(2x)}{2}}$

$\cos^2(x) = \dfrac{1 + \cos(2x)}{2}$

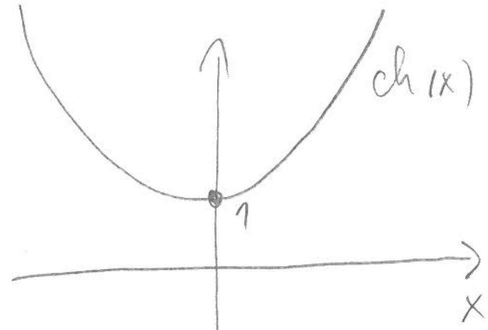
$$\mathrm{th}(x) = \frac{\mathrm{sh}(x)}{\mathrm{ch}(x)} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\mathrm{cth}(x) = \frac{\mathrm{ch}(x)}{\mathrm{sh}(x)} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$



É.T: $(-\infty, \infty)$

É.K: $(-\infty, \infty)$

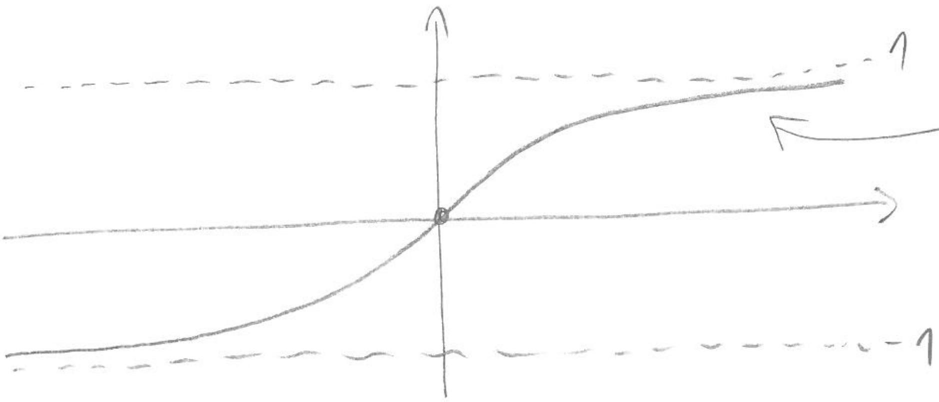


É.T: $(-\infty, \infty)$

É.K: $[1, \infty)$

sh(x) PÁRATLAN

ch(x) PÁROS



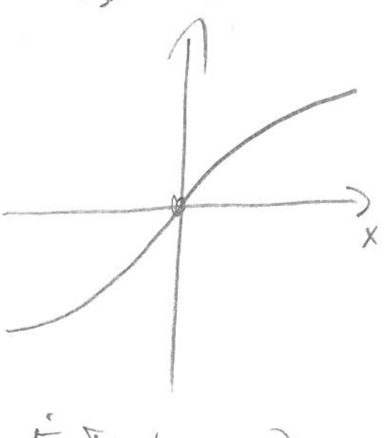
th(x)

É.T.: $(-\infty, \infty)$

É.K: $(-1, 1)$

AREA FÜGGVÉNYEK = HIPERBOLIKUS F.V.EK INVERZEI

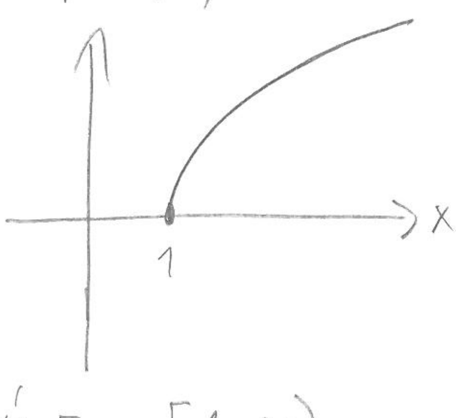
arsh(x)



É.T: $(-\infty, \infty)$

É.K: $(-\infty, \infty)$

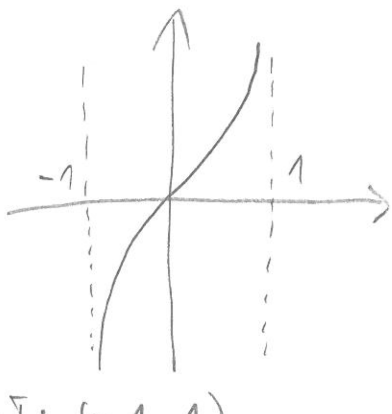
arch(x)



É.T.: $[1, \infty)$

É.K.: $[0, \infty)$

arth(x)



É.T: $(-1, 1)$

É.K: $(-\infty, \infty)$

ÁLLÍTÁS: $\boxed{\operatorname{arsh}(x) = \ln\left(x + \sqrt{x^2+1}\right)}$

BIZ: $f(x) = \operatorname{sh}(x) \quad f^{-1}(x) = ?$

$$y = \operatorname{sh}(x) = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} \iff y \cdot 2e^x = e^{2x} - 1$$

$$\iff (e^x)^2 - 2y \cdot (e^x) - 1 = 0 \iff u^2 - 2y \cdot u - 1 = 0$$

$$\Rightarrow u_{1,2} = \frac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1}$$

NEKÜNK A POZITÍV GYÖK KELL, mert $0 < e^x = u$

$$e^x = y + \sqrt{y^2+1} \Rightarrow x = \boxed{\ln\left(y + \sqrt{y^2+1}\right) = f^{-1}(y)}$$

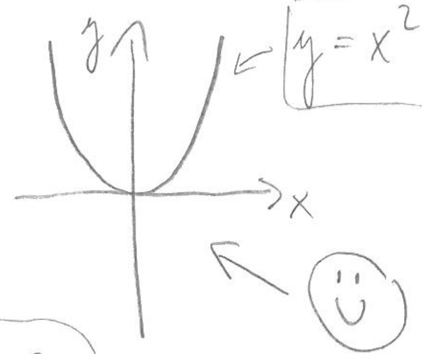
HASONLÓAN: $\operatorname{arch}(x) = \ln\left(x + \sqrt{x^2-1}\right)$

$$\operatorname{arth}(x) = \frac{1}{2} \cdot \ln\left(\frac{1+x}{1-x}\right)$$

GÖRBÉK A SÍKBAN:

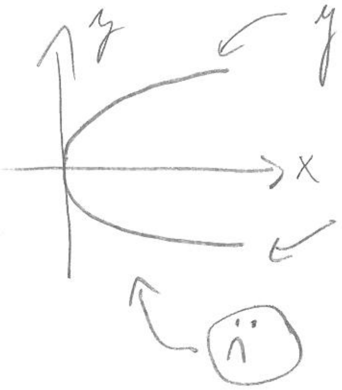
SOKFÉLEKÉPP MEGADHATÓAK:

1) EXPLICIT FÜGGVÉNY GRAFIKONJAKÉNT: $\boxed{y = f(x)}$



$\boxed{y = x^2}$ ← PARABOLA

DE EZ NEM MINDÍG MEGY:

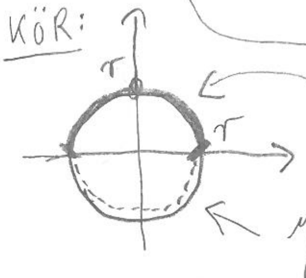


$y = \sqrt{x}$

$y = -\sqrt{x}$

KÉT FÜGGVÉNY GRAFIKONJA EGYÜTT = GÖRBE

KÖR:



$y = \sqrt{r^2 - x^2}$

$y = -\sqrt{r^2 - x^2}$

2) IMPLICIT MÓDON:

GÖRBE = AZON $P(x,y)$ PONTOK ÖSSZESSÉGE, AMIRE TELJESÜL, HOGY $\boxed{F(x,y) = 0}$

PL: $F(x,y) = x^2 - y$; $F(x,y) = x - y^2$

PL: $P_0(x_0, y_0)$ KÖZÉPPONTÚ r SUGARÚ KÖR.

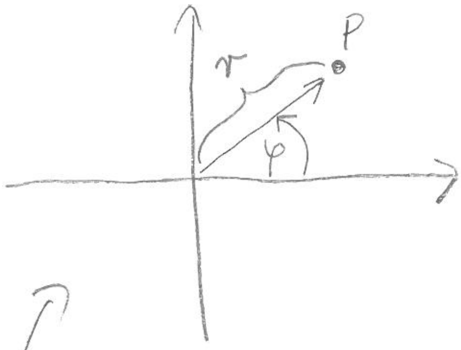
$P(x,y)$ RAJTA VAN, HA $|\overrightarrow{P_0P}| = r$, AZAZ

HA $\sqrt{(x-x_0)^2 + (y-y_0)^2} = r$, AZAZ HA

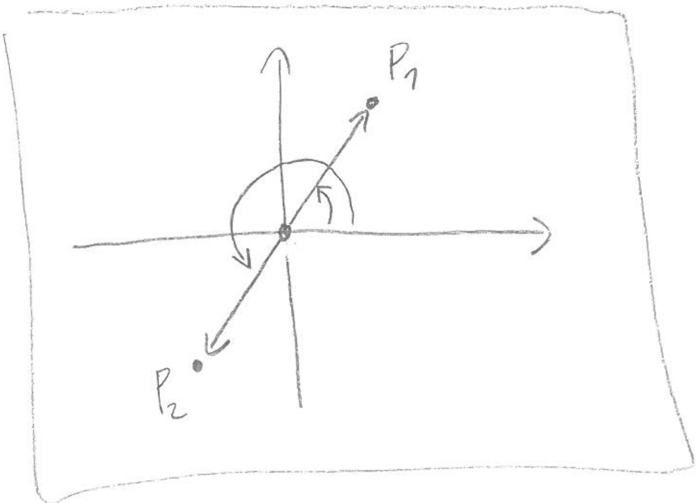
$$F(x,y) = (x-x_0)^2 + (y-y_0)^2 - r^2 = 0$$

46. OLDAL

(3) POLÁRKOORDINÁTÁKKAL:



PL.:



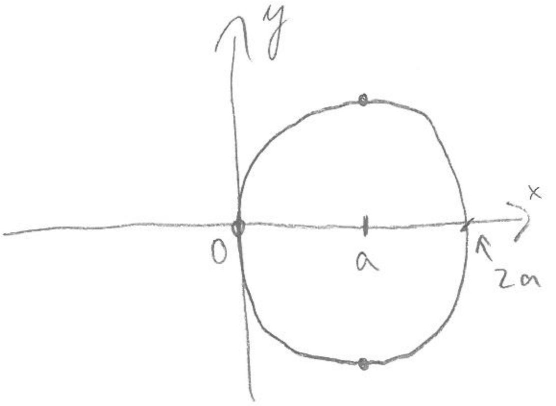
$\boxed{\text{r NEGATÍV IS LEHET!}}$

$$P_1\left(x=2,\ y=2\cdot\sqrt{3}\right) \longrightarrow P_1\left(\varphi=\frac{\pi}{3},\ r=4\right)$$

$$P_2\left(x=-2,\ y=-2\cdot\sqrt{3}\right) \longrightarrow P_2\left(\varphi=\frac{4}{3}\pi,\ r=4\right) = P_2\left(\varphi=\frac{1}{3}\pi,\ r=-4\right)$$

ORIGÓ KÖZÉPPONTÚ a SUGARÚ KÖR KÉPLETE POLÁRKOORDINÁTA-RENDSZERBEN: $\boxed{r=a}$

PL.: IMPLICIT EGYENLET: $(x-a)^2+y^2=a^2$



POLÁR:

$$(r\cdot\cos(\varphi)-a)^2+(r\cdot\sin(\varphi))^2=a^2$$

$$\boxed{r^2\cdot\cos^2\varphi-2ar\cos\varphi+a^2+r^2\cdot\sin^2\varphi=a^2}$$

$$r^2\cdot\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}-2ar\cos\varphi=0 \implies \boxed{r=2a\cos\varphi}$$

EXPLICIT POLÁR EGYENLET 47. OLDAL