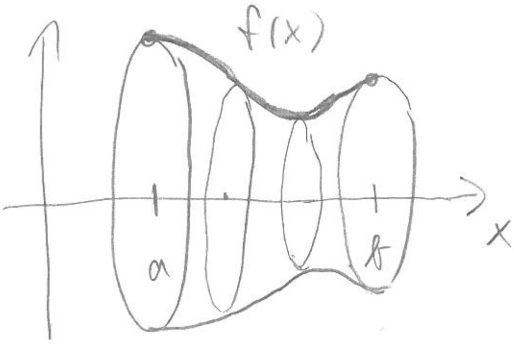
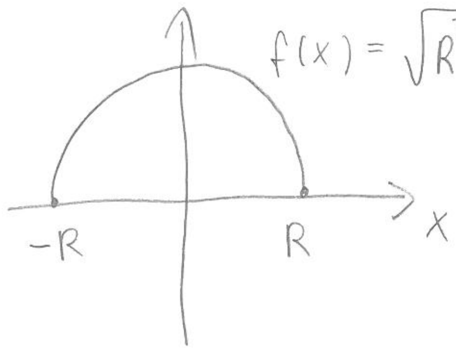
FORGÁSTEST FELSZÍNE:



$$\boxed{F = 2\pi \cdot \int_a^b f(x) \cdot \sqrt{1+(f'(x))^2}\, dx}$$



PL: R SUGARÚ GÖMB:

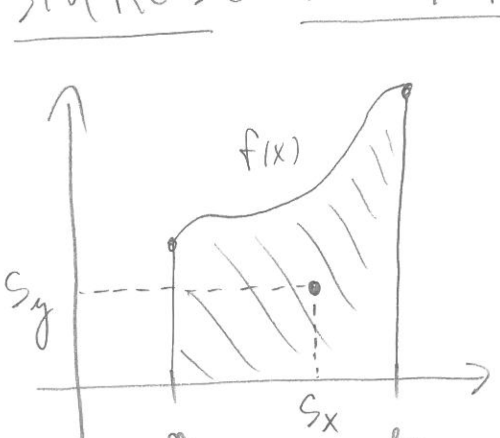
$$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{R^2-x^2}} \cdot (-2x) = \frac{-x}{\sqrt{R^2-x^2}}$$

$$\sqrt{1+(f'(x))^2} = \sqrt{1+\frac{x^2}{R^2-x^2}} = \sqrt{\frac{R^2}{R^2-x^2}} = \frac{R}{\sqrt{R^2-x^2}}$$

$$f(x) \cdot \sqrt{1+(f'(x))^2} = R$$

$$F = 2\pi \cdot \int_{-R}^{R} R\, dx = 2\pi \cdot R \cdot \underbrace{\int_{-R}^{R} 1\, dx}_{2R} = 4\pi \cdot R^2 \checkmark$$

SÍKRÉSZ SÚLYPONTJA:



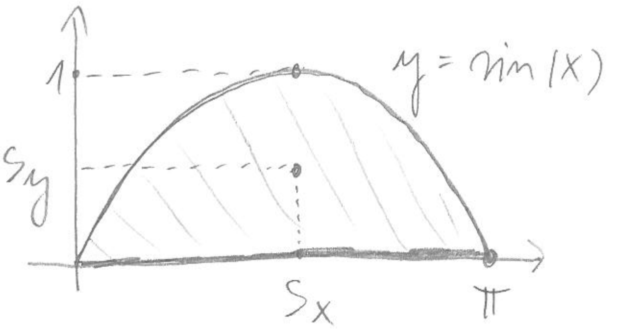
$$\left.\begin{aligned} T &= \int_a^b f(x)\, dx \\ M_x &= \frac{1}{2}\int_a^b f^2(x)\, dx \\ M_y &= \int_a^b x\, f(x)\, dx \end{aligned}\right\} \Rightarrow$$

$$\boxed{S_x = \frac{M_y}{T}}$$

$$\boxed{S_y = \frac{M_x}{T}}$$

162. OLDAL

PL: MIK AZ $\mathcal{R}$ SÍKIDOM SÚLYPONTJÁNAK KOORDINÁTÁI? AHOL $\mathcal{R} =$



$$T = \int_0^\pi \sin(x)\,dx = \Big[-\cos(x)\Big]_0^\pi = \cos(0) - \cos(\pi) = 2$$

PARCIÁLIS INTEGRÁLÁS

$$S_x = \left(\frac{\int_0^\pi x\cdot\sin(x)\,dx}{T}\right) = \frac{1}{2}\cdot\left(\Big[x\cdot(-\cos(x))\Big]_0^\pi - \int_0^\pi -\cos(x)\,dx\right) =$$

$$= \frac{1}{2}\cdot\left(\pi\cdot(-\cos(\pi)) - 0\cdot(-\cos(0)) + \Big[\sin(x)\Big]_0^\pi\right) =$$

$$= \frac{1}{2}\cdot\left(\pi\cdot 1 - 0 + \overbrace{\sin(\pi)}^{0} - \overbrace{\sin(0)}^{0}\right) = \pi/2$$

$$S_y = \left(\frac{\frac{1}{2}\cdot\int_0^\pi \sin^2(x)\,dx}{T}\right) = \frac{1}{4}\cdot\int_0^\pi \frac{1}{2}\cdot\big(1-\cos(2x)\big)\,dx =$$

$$= \frac{1}{4}\cdot\left(\int_0^\pi \frac{1}{2}\,dx - \frac{1}{2}\int_0^\pi \cos(2x)\,dx\right) = \frac{1}{4}\cdot\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{8}$$

MEGJ: $\boxed{S_x = \frac{\pi}{2}}$: SZIMMETRIA MIATT KÖZÉPEN VAN

$\boxed{S_y = \frac{\pi}{8} < \frac{1}{2}}$: ALUL SZÉLESEBB A SÍKIDOM

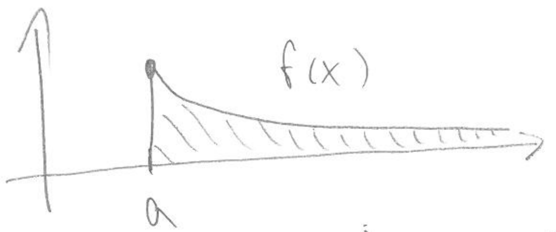
163. OLDAL

IMPROPRIUS INTEGRÁL:

(SZÓ SZERINT: „HELYTELEN"

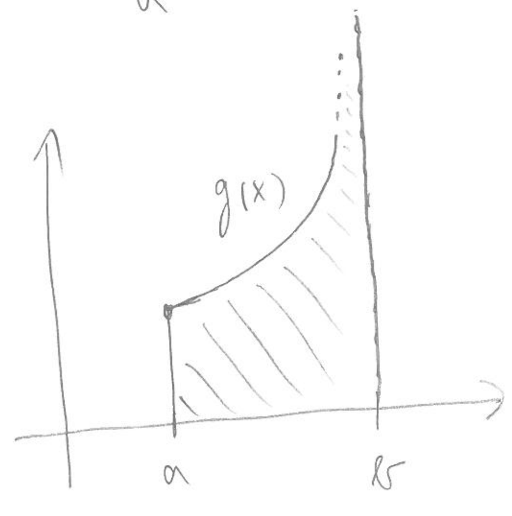
OLYAN SÍKIDOMOK TERÜLETE, AMIK VALAMILYEN IRÁNYBAN VÉGTELENEK.

KÉT FAJTA VAN:

1. 

$$\int_a^\infty f(x)\,dx$$

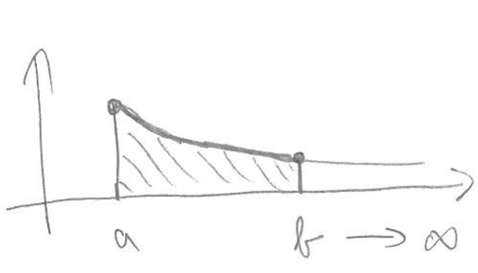
VÍZSZINTESEN TART ∞-HEZ

2. 

$$\int_a^b g(x)\,dx, \text{ DE } \lim_{x\to b}|f(x)|=\infty$$

FÜGGŐLEGESEN TART ∞-HEZ

HOGY SZÁMOLJUK KI ŐKET?

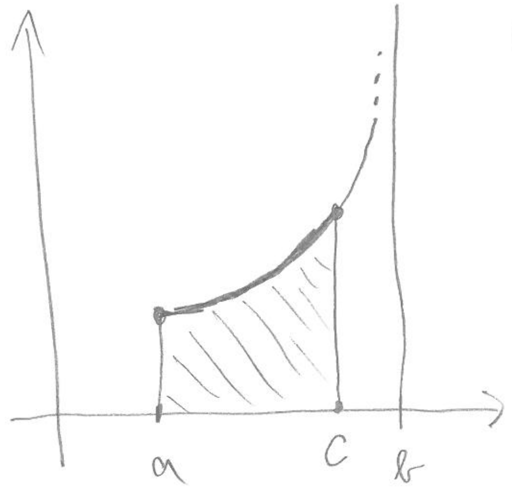
1. 

DEFINÍCIÓ:

$$\boxed{\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx}$$

164. OLDAL

2



DEF:

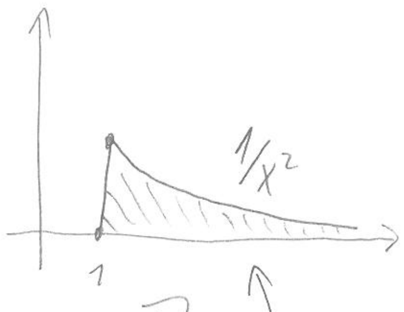
$$\int_a^b f(x)\,dx = \lim_{c \to b_-} \int_a^c f(x)\,dx$$

C BALRÓL KONVERGÁL b-HEZ

PÉLDÁK: VÍZSZINTES:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx = \lim_{b\to\infty} \left[-\frac{1}{x}\right]_1^b =$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b} - (-1)\right) = -0 + 1 = 1$$

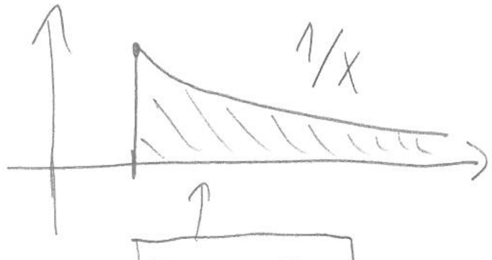


NEM KORLÁTOS TARTOMÁNY, DE A TERÜLETE VÉGES!

$\boxed{T=1}$

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \left[\ln(x)\right]_1^b =$$

$$\lim_{b\to\infty} \left(\ln(b) - \ln(1)\right) = +\infty$$



$\boxed{T=+\infty}$

165. OLDAL

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\left[2\cdot\sqrt{x}\right]_1^b =$$

$$= \lim_{b\to\infty} 2\cdot\sqrt{b}-2 = +\infty$$

<u>TÉTEL</u>: HA $\boxed{a>0}$, AKKOR

$$\int_a^\infty \frac{1}{x^p}\,dx = \begin{cases} \underline{\text{VÉGES}}, \text{ HA } \boxed{p>1} \\ \underline{\text{VÉGTELEN}}, \text{ HA } \boxed{p\le 1} \end{cases}$$

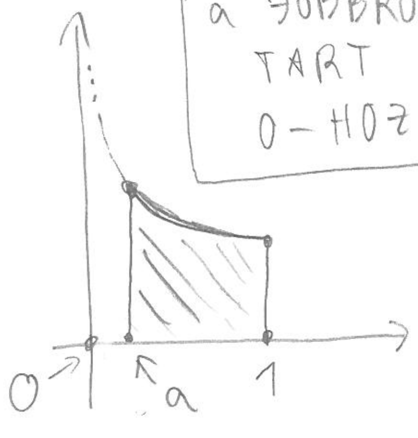
<u>BIZ</u>: HASONLÓAN AZ ELŐZŐ 3 PÉLDÁHOZ, AMIK A $\boxed{p=2}$ $\boxed{p=1}$ $\boxed{p=\frac{1}{2}}$ ESETEK VOLTAK.

<u>DEF</u>: $\int_a^\infty f(x)\,dx < \infty \Leftarrow$ $\boxed{\text{IMPROPRIUS INTEGRÁL KONVERGENS}}$

$\int_a^\infty f(x)\,dx = +\infty \Leftarrow$ $\boxed{\text{IMP. INT. DIVERGENS}}$

<u>PL</u>: FÜGGŐLEGES: $$\int_0^1 \frac{1}{x^2}\,dx = \lim_{a\to 0_+}\int_a^1 \frac{1}{x^2}\,dx =$$

$\boxed{\text{a JOBBRÓL TART 0-HOZ}}$



$$= \lim_{a\to 0_+}\left[-\frac{1}{x}\right]_a^1 = \lim_{a\to 0_+}\left(-1+\frac{1}{a}\right) =$$

$= +\infty$ DIVERGENS!

$\boxed{\text{166. OLDAL}}$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0+} \int_a^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0+} \left[2\cdot\sqrt{x}\right]_a^1 =$$

$$= \lim_{a\to 0+} \left(2\cdot\sqrt{1} - 2\cdot\sqrt{a}\right) = 2$$ KONVERGENS!

$$\int_0^1 \frac{1}{x}\,dx = \lim_{a\to 0+} \int_a^1 \frac{1}{x}\,dx = \lim_{a\to 0+} \left[\ln(x)\right]_a^1 =$$

$$= \lim_{a\to 0+} \left(\ln(1) - \ln(a)\right) = \lim_{a\to 0+} \ln\left(\frac{1}{a}\right) = +\infty$$ DIVERGENS

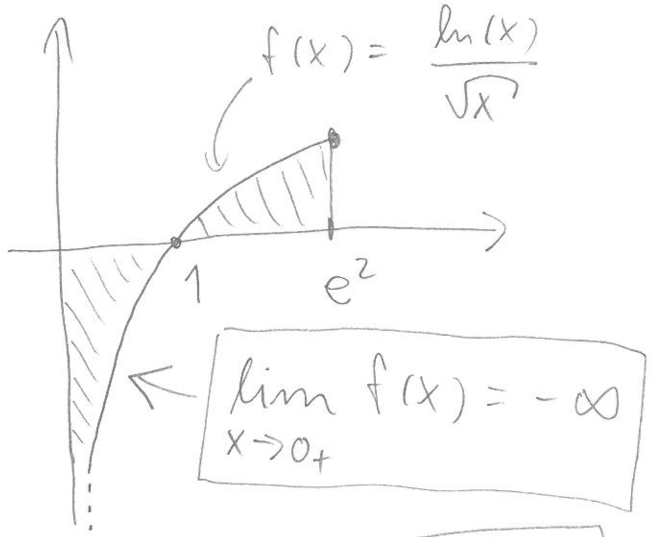
TÉTEL: HA $\boxed{a>0}$:

$$\int_0^a \frac{1}{x^p}\,dx = \begin{cases} \text{VÉGES, HA } \boxed{p<1} \\ \text{VÉGTELEN, HA } \boxed{p\geq 1} \end{cases}$$

BIZ: HASONLÓAN AZ ELŐZŐ 3 PÉLDÁHOZ, AMIK A $\boxed{p=2}$ $\boxed{p=1/2}$ $\boxed{p=1}$ ESETEK VOLTAK.

PL: $\displaystyle\int_0^{e^2} \frac{\ln(x)}{\sqrt{x}}\,dx = \star$

VALÓBAN IMPROPRIUS:



ELŐSZÖR SZÁMOLJUK KI A HATÁROZATLAN INTEGRÁLT:

$$\int \frac{\ln(x)}{\sqrt{x}}\,dx = \int \underbrace{\ln(x)}_{u} \cdot \underbrace{\frac{1}{\sqrt{x}}}_{v'}\,dx =$$

PARCIÁLIS INTEGRÁLÁS:

$u(x) = \ln(x) \Rightarrow u'(x) = 1/x$

$v'(x) = \frac{1}{\sqrt{x}} \Rightarrow v(x) = \int \frac{1}{\sqrt{x}}\,dx = 2 \cdot \sqrt{x}$

$$= \underbrace{\ln(x)}_{u} \cdot \underbrace{2 \cdot \sqrt{x}}_{v} - \int \underbrace{1/x}_{u'} \cdot 2 \cdot \sqrt{x}\,dx =$$

$$= 2 \cdot \sqrt{x} \cdot \ln(x) - 2 \cdot \underbrace{\int x^{-1/2}\,dx}_{2 \cdot x^{1/2}} = 2 \cdot \sqrt{x} \cdot (\ln(x) - 2)$$

$$\star = \lim_{a \to 0+} \int_a^{e^2} \frac{\ln(x)}{\sqrt{x}}\,dx = \lim_{a \to 0+} \Big[2 \cdot \sqrt{x} \cdot (\ln(x) - 2) \Big]_a^{e^2} =$$

$$= \underbrace{2 \cdot \sqrt{e^2} \cdot (\ln(e^2) - 2)}_{2 \cdot e \cdot (2-2) = 0} - \underbrace{\lim_{a \to 0+} 2 \cdot \sqrt{a} \cdot (\ln(a) - 2)}_{= 0} = 0$$

← MIÉRT?

168. OLDAL

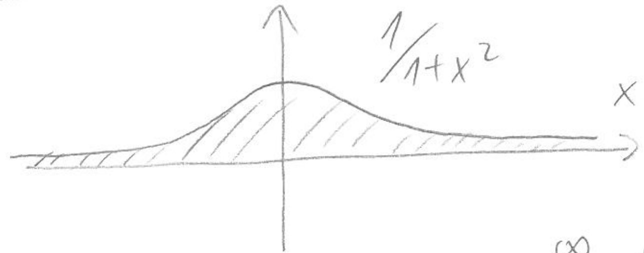
$$\lim_{a\to 0+} \sqrt{a}\cdot(\ln(a)-2) = \lim_{a\to 0+} \frac{\ln(a)-2}{a^{-1/2}} =$$

$= \frac{\infty}{\infty}$ TÍPUSÚ LIMESZ: L'HOSPITAL-SZABÁLY!

$$= \lim_{a\to 0+} \frac{(\ln(a)-2)'}{(a^{-1/2})'} = \lim_{a\to 0+} \frac{a^{-1}}{-\frac{1}{2}\cdot a^{-3/2}} =$$

$$= \lim_{a\to 0+} (-2)\cdot\sqrt{a} = 0$$

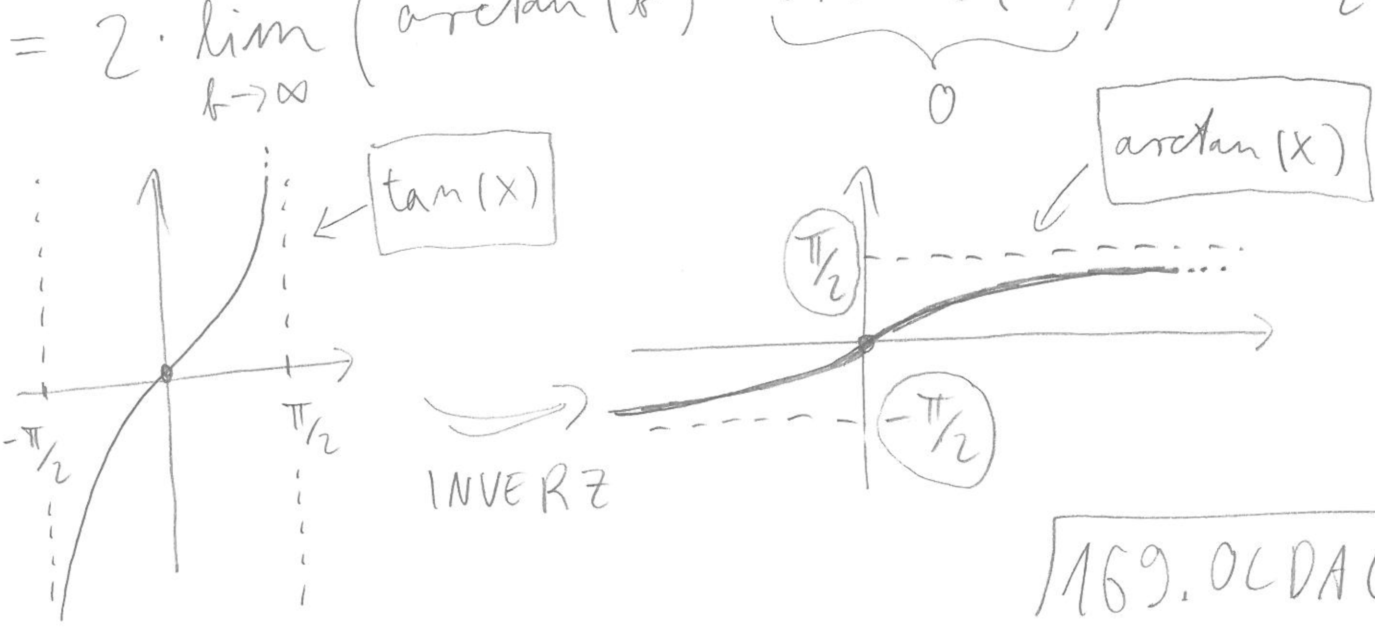
PL: $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = ?$



$$? = \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{\infty} \frac{dx}{1+x^2} = \text{SZIMMETRIA} = 2\cdot\int_{0}^{\infty}\frac{dx}{1+x^2} =$$

$$= 2\cdot \lim_{b\to\infty} \int_0^b \frac{dx}{1+x^2} = 2\cdot\lim_{b\to\infty} \Big[\arctan(x)\Big]_0^b =$$

$$= 2\cdot\lim_{b\to\infty}\Big(\arctan(b) - \underbrace{\arctan(0)}_{0}\Big) = 2\cdot\frac{\pi}{2} = \pi$$



169. OLDAL