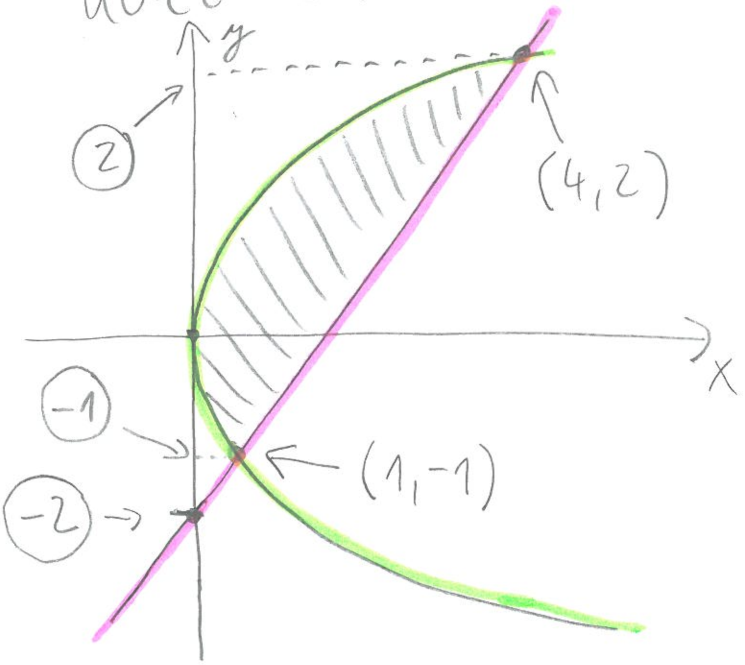
MI AZ $\boxed{x=y^2}$ ÉS AZ $\boxed{x=y+2}$ GÖRBÉK KÖZÉ ESŐ SÍKIDOM TERÜLETE?



90°-AL ELFORGATVA KELL NÉZNI!

A METSZÉSPONTOK y-KOORDINÁTÁJA MEGOLDJA AZ

$$\boxed{y^2 = y+2}$$

EGYENLETET

$$y_{1,2} = \frac{1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2}$$

$\boxed{y_1 = -1}$ $\boxed{y_2 = 2}$

$\boxed{x_1 = 1}$ $\boxed{x_2 = 4}$

TERÜLET =

$$\int_{-1}^{2} \left((y+2) - y^2\right) dy =$$

$$= \left[\frac{1}{2}\cdot y^2 + 2y - \frac{1}{3}\cdot y^3\right]_{-1}^{2} =$$

$$= \left(\frac{1}{2}\cdot 4 + 2\cdot 2 - \frac{1}{3}\cdot 8\right) - \left(\frac{1}{2}\cdot 1 - 2 + \frac{1}{3}\right) = 4.5$$

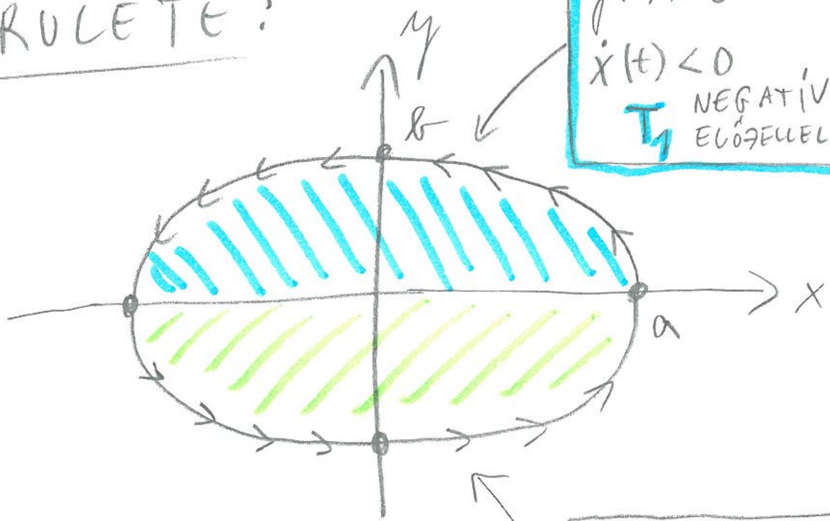
PARAMÉTERESEN MEGADOTT GÖRBE ALATTI TERÜLET:

$$\boxed{\begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned}} \qquad \boxed{T = \int_{t_1}^{t_2} y(t)\cdot \dot{x}(t)\,dt}$$

(AZ X TENGELY ÉS A GÖRBE KÖZTI $\overbrace{\text{TERÜLET}}^{\text{ELŐJELES}}$)

PL: ELLIPSZIS TERÜLETE:

$$\boxed{\begin{aligned} x(t) &= a\cdot \cos(t) \\ y(t) &= b\cdot \sin(t) \end{aligned}}$$



$y(t) > 0$

$\dot{x}(t) < 0$

T_1 NEGATÍV ELŐJELLEL

$y(t) < 0$

$\dot{x}(t) > 0$

T_2 NEGATÍV ELŐJELLEL

$$T_1 = -\int_0^{\pi} y(t)\cdot \dot{x}(t)\,dt$$

$$T_2 = -\int_{\pi}^{2\pi} y(t)\cdot \dot{x}(t)\,dt$$

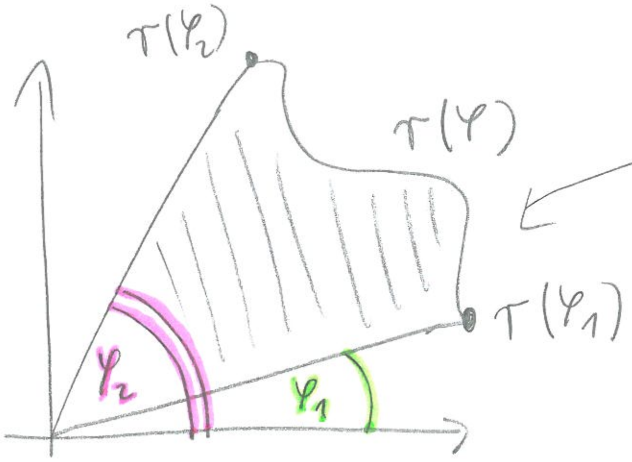
$$T = T_1 + T_2 = -\int_0^{2\pi} y(t)\cdot \dot{x}(t)\,dt =$$

$$-\int_0^{2\pi} (b\cdot \sin(t))\cdot(-a\cdot \sin(t))\,dt = a\cdot b\cdot \int_0^{2\pi} \sin^2(t)\,dt =$$

$$= \boxed{125.\text{ OLD}} = a\cdot b\cdot \left[\frac{1}{2}t - \frac{1}{4}\cdot \sin(2t)\right]_0^{2\pi} = a\cdot b\cdot \pi$$

154. OLDAL

POLÁRKOORDINÁTÁKKAL MEGADOTT SÍKIDOM TERÜLETE:



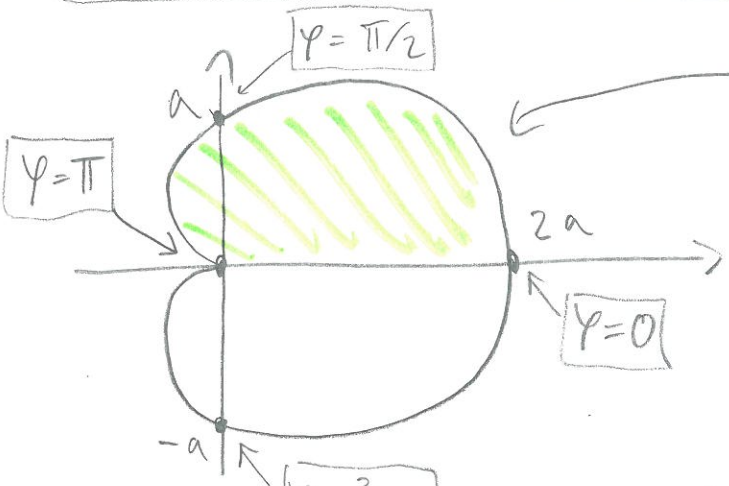
SZEKTOR:

$$\boxed{T = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} r^2(\varphi)\,d\varphi}$$

PL: KARDIOID (SZÍV ALAKÚ)

$$\boxed{r(\varphi) = a\cdot(1+\cos(\varphi))}$$

FELSŐ FELE: $\boxed{0 \le \varphi \le \pi}$



$$T = \frac{1}{2}\int_0^{\pi} r^2(\varphi)\,d\varphi =$$

$$= \frac{1}{2}\int_0^{\pi} a^2\cdot(1+\cos(\varphi))^2\,d\varphi =$$

$$= \frac{1}{2}a^2\cdot\int_0^{\pi}\left(1+2\cos(\varphi)+\cos^2(\varphi)\right)d\varphi =$$

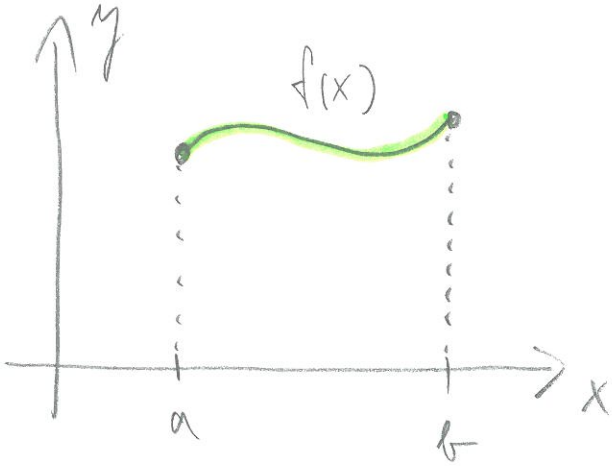
$$= \frac{1}{2}a^2\cdot\int_0^{\pi}\left(1+2\cdot\cos(\varphi)+\frac{1+\cos(2\varphi)}{2}\right)d\varphi =$$

$$= \frac{1}{2}a^2\cdot\left[\frac{3}{2}\varphi+2\cdot\sin(\varphi)+\frac{\sin(2\varphi)}{4}\right]_0^{\pi} = \frac{1}{2}a^2\cdot\frac{3}{2}\pi$$

155. OLDAL

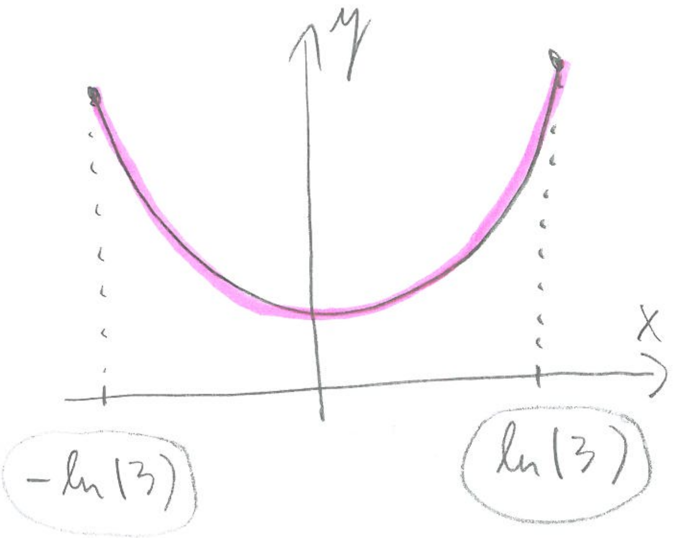
SÍKGÖRBE ÍVHOSSZA:

$$\boxed{S = \int_a^b \sqrt{1+\left(f'(x)\right)^2}\,dx}$$



PL: LÁNCGÖRBE: $\boxed{y = ch(x)}$

$$\boxed{\begin{aligned} a &= -\ln(3) \\ b &= \ln(3) \end{aligned}}$$



$$S = \int_{-\ln(3)}^{\ln(3)} \sqrt{1+sh^2(x)}\,dx =$$

$$\boxed{1+sh^2(x) = ch^2(x)}$$

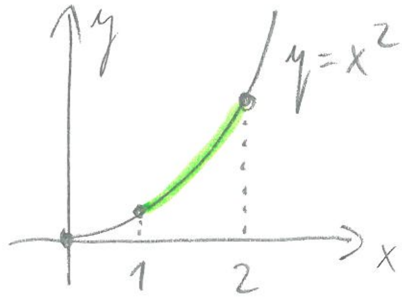
$$= \int_{-\ln(3)}^{\ln(3)} ch(x)\,dx = \Big[sh(x)\Big]_{-\ln(3)}^{\ln(3)} =$$

$$= \left(\frac{e^{\ln(3)} - e^{-\ln(3)}}{2}\right) - \left(\frac{e^{-\ln(3)} - e^{-(-\ln(3))}}{2}\right) =$$

$$= 2\cdot\left(\frac{3-\frac{1}{3}}{2}\right) = \frac{8}{3}$$

PARABOLA ÍVHOSSZA:

$\boxed{f(x)=x^2}$ $\boxed{a=1}$ $\boxed{b=2}$



$$\int_1^2 \sqrt{1+(f'(x))^2}\,dx = \int_1^2 \sqrt{1+(2x)^2}\,dx =$$

LÁSD 143. OLDAL:

$2x = sh(u)$

$\frac{dx}{du} = \frac{1}{2} ch(u)$

$dx = \frac{1}{2} ch(u)\,du$

$\sqrt{1+(2x)^2} =$

$\sqrt{1+(sh(u))^2} =$

$\sqrt{ch(u)^2} =$

$ch(u)$

$$= \int_{arsh(2)}^{arsh(4)} ch(u)\,\frac{1}{2}\,ch(u)\,du =$$

$\boxed{x=1} \Rightarrow$

$2\cdot 1 = sh(u) \Rightarrow$

$u = arsh(2)$

$\boxed{x=2} \Rightarrow$

$2\cdot 2 = sh(u) \Rightarrow$

$u = arsh(4)$

$$= \frac{1}{2}\int_{arsh(2)}^{arsh(4)} ch^2(u)\,du = \frac{1}{2}\int_{a(2)}^{a(4)} \frac{ch(2u)+1}{2}\,du$$

$$= \frac{1}{4}\left[\frac{1}{2} sh(2u) + u\right]_{a(2)}^{a(4)} =$$

$$= \boxed{\begin{aligned} sh(2u) &= 2\,sh(u)\cdot ch(u) = \\ &= 2\,sh(u)\cdot\sqrt{1+sh^2(u)} \end{aligned}}$$

$$= \frac{1}{8}\cdot\left(2\cdot 4\cdot\sqrt{1+4^2} - 2\cdot 2\cdot\sqrt{1+2^2}\right)$$

$$+ \frac{1}{4}\cdot\left(arsh(4) - arsh(2)\right) = 3.1678$$

157. OLDAL

PARAMÉTERES GÖRBE ÍVHOSSZA:

$$\boxed{\begin{matrix} x = x(t) \\ y = x(t) \end{matrix}} \Longrightarrow \boxed{S = \int_{t_1}^{t_2} \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}\, dt}$$

PL: KÖR: $\boxed{x(t) = r \cdot \cos(t)}$ $\boxed{y(t) = r \cdot \sin(t)}$

$$S = \int_0^{2\pi} \sqrt{(r \cdot (-\sin(t)))^2 + (r \cdot \cos(t))^2}\, dt =$$

$$= \int_0^{2\pi} r \cdot \sqrt{\sin^2(t) + \cos^2(t)}\, dt = \int_0^{2\pi} r\, dt = r \cdot 2\pi$$

POLÁRKOORDINÁTÁKKAL MEGADOTT GÖRBE ÍVHOSSZA:

$$\boxed{r = r(\varphi)} \Longrightarrow \boxed{\begin{matrix} x(\varphi) = r(\varphi) \cdot \cos(\varphi) \\ y(\varphi) = r(\varphi) \cdot \sin(\varphi) \end{matrix}} \Longrightarrow \boxed{\varphi = t}$$

$$\Longrightarrow \boxed{\begin{matrix} x(t) = r(t) \cdot \cos(t) \\ y(t) = r(t) \cdot \sin(t) \end{matrix}} \Longrightarrow \begin{matrix} \dot{x}(t) = \dot{r}(t) \cdot \cos(t) - r(t) \cdot \sin(t) \\ \dot{y}(t) = \dot{r}(t) \cdot \sin(t) + r(t) \cdot \cos(t) \end{matrix}$$

$$\Longrightarrow \sqrt{(\dot{x}(t))^2 + (\dot{y}(t))^2} = [\ldots] = \sqrt{(\dot{r}(\varphi))^2 + (r(\varphi))^2}$$

$$\Longrightarrow \boxed{S = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2(\varphi) + (r'(\varphi))^2}\, d\varphi}$$

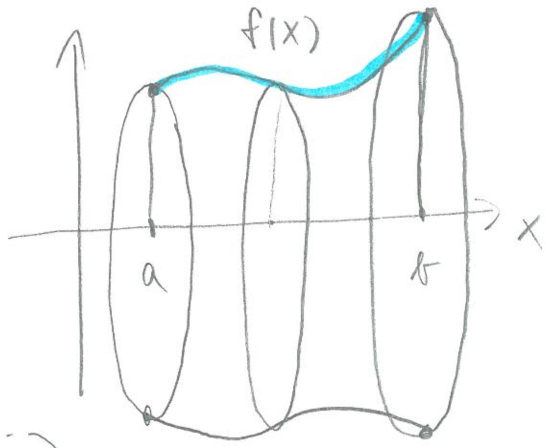
158. OLDAL

PL: KÖR KERÜLETE: $\boxed{r(\varphi)\equiv r}$ $\boxed{0\le\varphi\le 2\pi}$

$$\Rightarrow \boxed{r'(\varphi)\equiv 0} \Rightarrow \sqrt{r^2+(r')^2}\equiv r \Rightarrow$$

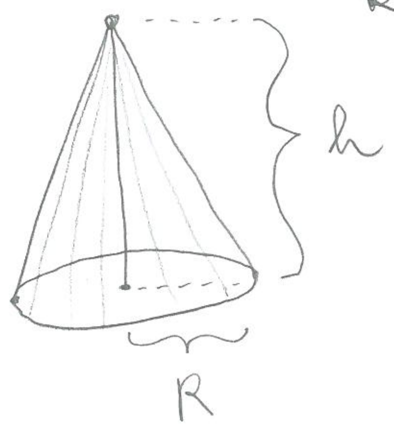
$$S=\int_0^{2\pi} r\,d\varphi = r\cdot 2\pi$$

FORGÁSTEST TÉRFOGATA:

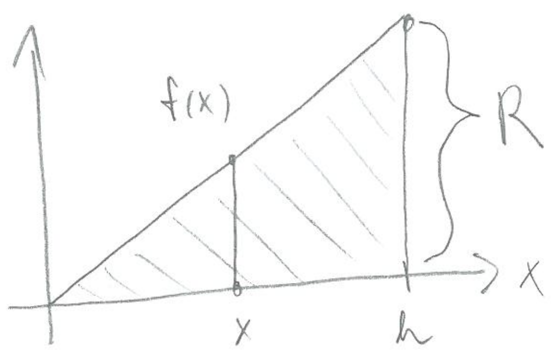


FORGÁSTENGELY: X TENGELY

$$\boxed{V=\pi\cdot\int_a^b f^2(x)\,dx}$$



PL: KÚP:



$$\boxed{f(x)=\frac{R}{h}\cdot x}$$

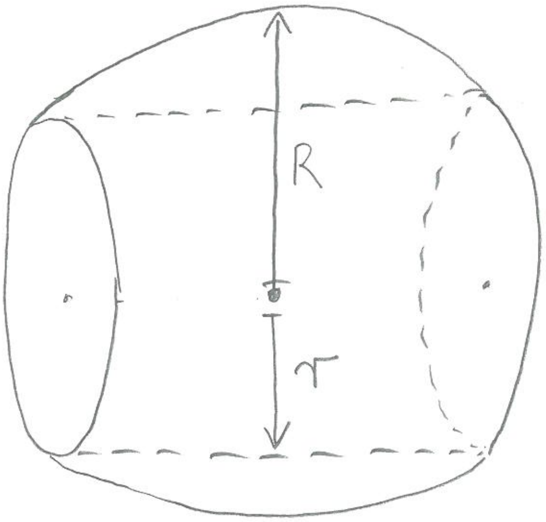
$\boxed{f(0)=0}$

$\boxed{f(h)=R}$

$$V=\pi\cdot\int_0^h f^2(x)\,dx=\pi\cdot\int_0^h\left(\frac{Rx}{h}\right)^2dx=$$

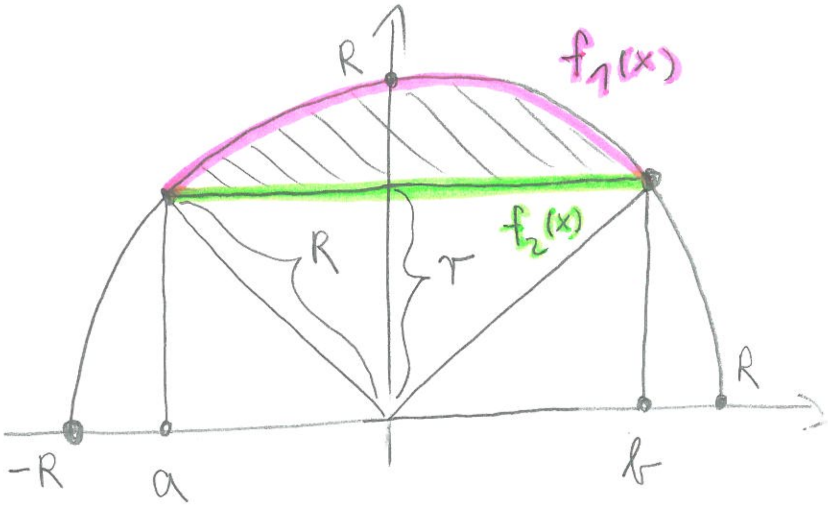
$$\pi\cdot\frac{R^2}{h^2}\cdot\left[\frac{1}{3}x^3\right]_0^h=\frac{\pi\cdot R^2}{3}\cdot h$$

PL: GÖMB, AMIBE EGY KÖR ALAKÚ ALAGUTAT FÚRTAK: $\boxed{r \le R}$



$$\boxed{V = ?}$$

EZT A SÍKIDOMOT KELL MEGFORGATNI AZ X-TENGELY KÖRÜL:



PITHAGORASZ $\Rightarrow$ $\boxed{a = -\sqrt{R^2 - r^2}}$ $\boxed{b = \sqrt{R^2 - r^2}}$

$$V = V_1 - V_2 =$$

$$= \pi \cdot \int_a^b f_1^2(x)\,dx - \pi \int_a^b f_2^2(x)\,dx = \pi \cdot \int_a^b \left(f_1^2(x) - f_2^2(x)\right) dx =$$

$$= \text{SZIMMETRIA} = 2 \cdot \pi \cdot \int_0^{\sqrt{R^2 - r^2}} \left(\sqrt{R^2 - x^2}^{\,2} - r^2 \right) dx = @$$

$$\boxed{\begin{aligned} f_1(x) &= \sqrt{R^2 - x^2} \\ f_2(x) &\equiv r \end{aligned}}$$

160. OLDAL

$$@ = 2\pi \cdot \int_0^{\sqrt{R^2-r^2}} \left(R^2 - r^2 - x^2\right) dx =$$

$$= 2\pi \cdot \left[\left(R^2 - r^2\right) \cdot x - \frac{1}{3} x^3 \right]_0^{\sqrt{R^2-r^2}} =$$

$$= 2\pi \cdot \left(\left(R^2-r^2\right) \cdot \sqrt{R^2-r^2} - \frac{1}{3} \cdot \sqrt{R^2-r^2}^{\,3} \right)$$

$$= \frac{4}{3} \pi \cdot \left(R^2 - r^2\right)^{3/2}$$

HA $\boxed{r = 0}$, AKKOR AZ ALAGÚT 0 SUGARÚ, TEHÁT AZ R SUGARÚ GÖMB TÉRFOGATÁT KAPJUK:

$$V = \frac{4}{3} \pi \cdot \left(R^2\right)^{3/2} = \frac{4}{3} R^3 \cdot \pi$$