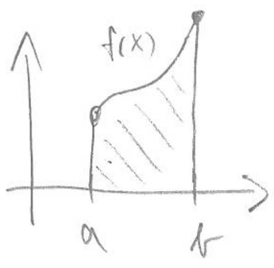
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n$$

AHOGY $n \to \infty$, $\Delta x = \frac{b-a}{n} \to 0$, A FELOSZTÁS EGYRE FINOMODIK, ÉS ÍGY R_n EGYRE PONTOSABBAN KÖZELÍTI A KERESETT TERÜLETET.

$\int_a^b f(x)\,dx$ = f HATÁROZOTT INTEGRÁLJA a-TÓL b-IG

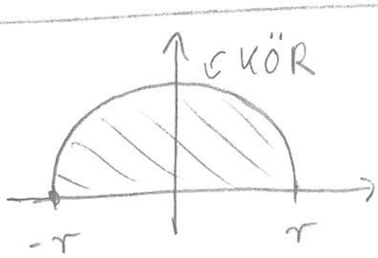
<u>MEGJ</u>: $\int_a^b f(x)\,dx$ NEM x FÜGGVÉNYE, MINT A HATÁROZATLAN INTEGRÁL $\int f(x)\,dx$!

$\int_a^b f(x)\,dx =$



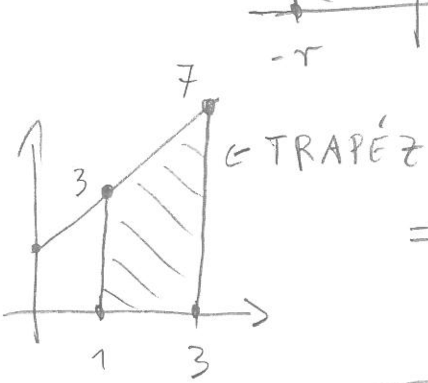
$= \int_a^b f(t)\,dt = \int_a^b f(u)\,du$ STB.

<u>PL</u>: $\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx =$



$= \frac{r^2 \cdot \pi}{2}$

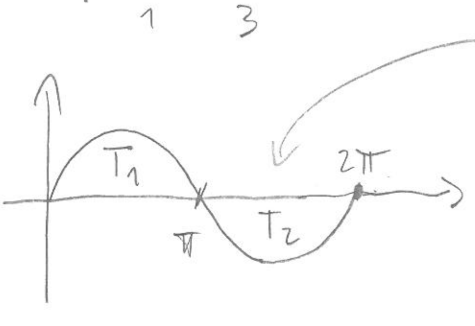
$\int_1^3 (2x+1)\,dx =$



$= (3-1)\cdot\left(\frac{3+7}{2}\right) = 10$

ELŐJELES TERÜLET

$\int_0^{2\pi} \sin(x)\,dx =$



$= T_1 - T_2 = 0$

146. OLDAL

HATÁROZOTT INTEGRÁL TULAJDONSÁGAI:

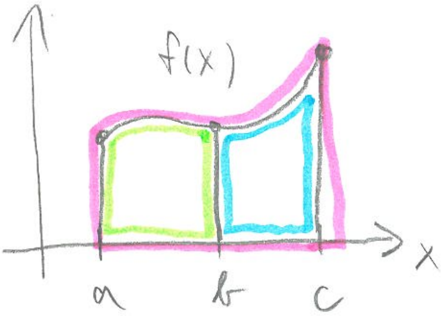
$$\boxed{\int_a^a f(x)\,dx = 0}$$

DEF: $$\boxed{\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx}$$

$$\boxed{\int_a^b c\cdot f(x)\,dx = c\cdot\int_a^b f(x)\,dx}$$

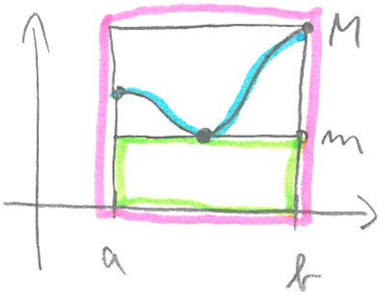
$$\boxed{\int_a^b \big(f(x)\pm g(x)\big)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx}$$

$$\boxed{\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx}$$



HA $m = \min\limits_{a\le x\le b} f(x)$ ÉS $M = \max\limits_{a\le x\le b} f(x)$, AKKOR

$$m\cdot(b-a) \le \int_a^b f(x)\,dx \le M\cdot(b-a)$$



KÖVETKEZMÉNY: INTEGRÁLSZÁMÍTÁS KÖZÉPÉRTÉK-TÉTELE:

HA $f:[a,b]\to\mathbb{R}$ FOLYTONOS, AKKOR VAN OLYAN $a\le x_0\le b$, HOGY $f(x_0) = \frac{1}{b-a}\cdot\int_a^b f(x)\,dx$

147. OLDAL

$\underline{\text{BIZ}}$: $m \le \underbrace{\frac{1}{b-a} \cdot \int_a^b f(x)\,dx}_{y_0} \le M$

WEIERSTRASS + BOLZANO (LÁSD 56-57. OLD):
VAN OLYAN $a \le x_0 \le b$, HOGY $f(x_0) = y_0$ ✓

<u>INTEGRÁLSZÁMÍTÁS</u> <u>ALAPTÉTELE</u>:

HA $\int f(x)\,dx = F(x) + C$ (AZAZ HA $F'(x) = f(x)$)

AKKOR $\boxed{\int_a^b f(x)\,dx = F(b) - F(a) = [F(x)]_a^b}$

↑ JELÖLÉS

$\underline{\text{BIZ}}$: $T(x) := \int_a^x f(u)\,du$ KORÁBBAN BE-

LÁTTUK, HOGY $T'(x) = f(x)$ (116. OLDAL)

TEHÁT $T(x) = F(x) + C$, $T(a) = \int_a^a f(u)\,du = 0$

$0 = F(a) + C \Rightarrow \boxed{C = -F(a)} \Longrightarrow$

$T(x) = F(x) - F(a) \Rightarrow \int_a^b f(x)\,dx = T(b) = F(b) - F(a)$

✓ 148. OLDAL

PL: $\int_1^3 (2x+1)\,dx = ?$ $f(x) = 2x+1$

$\int f(x)\,dx = x^2 + x + C$, $F(x) = x^2 + x$

$? = F(3) - F(1) = \left[x^2 + x\right]_1^3 = (3^2+3) - (1^2+1) =$

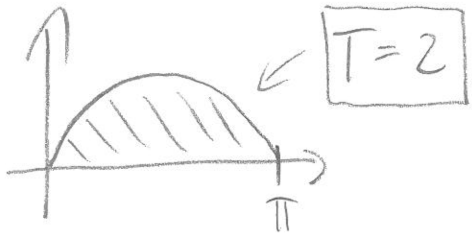
$= 12 - 2 = 10$ (LÁSD 146. OLDAL) ✓

PL: $\int_0^{2\pi} \sin(x)\,dx = \left[-\cos(x)\right]_0^{2\pi} = (-\cos(2\pi)) - (-\cos(0)) =$

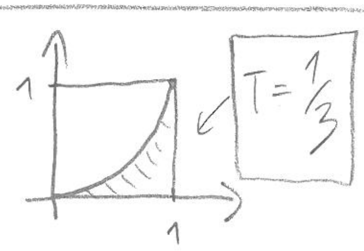
$= (-1) - (-1) = 0$ ✓ (LÁSD 146. OLDAL)

PL: $\int_0^{\pi} \sin(x)\,dx = \left[-\cos(x)\right]_0^{2\pi} = (-\cos(\pi)) - (-\cos(0)) =$

$= (-(-1)) - (-1) = 1 + 1 = 2$



PL: $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$



$\int_0^1 \sqrt{x}\,dx = \left[\frac{1}{1+1/2} \cdot x^{1+1/2}\right]_0^1 = \frac{2}{3}$

149. OLDAL

PL: $\int_2^3 \frac{x^2}{x^3+2}\,dx = \text{?}$ $\int \frac{x^2}{x^3+2}\,dx =$ | $u = x^3+2$ | $\frac{du}{3} =$ |
|---|---|
| $\frac{du}{dx} = 3x^2$ | $x^2 dx$ |

$$= \int \frac{1}{u}\frac{du}{3} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3+2| + C$$

$$\text{?} = \left[\frac{1}{3}\ln|x^3+2|\right]_2^3 = \frac{1}{3}\ln(3^3+2) - \frac{1}{3}\ln(2^3+2) =$$

$$= \frac{1}{3}\left(\ln(29) - \ln(10)\right) = \frac{\ln\left(\frac{29}{10}\right)}{3}$$

UGYANEZ MÁSKÉPP: HATÁROZOTT INTEGRÁL ESETÉN IS HELYETTESÍTHETEK, DE AKKOR AZ INTEGRÁL ALSÓ ÉS FELSŐ HATÁRÁT IS HELYETTESÍTENI KELL:

$\int_2^3 \frac{x^2}{x^3+2}\,dx =$

$u = x^3+2$	$\boxed{x=2} \Rightarrow u = 2^3+2 = 10$
$x^2 dx = \frac{1}{3}du$	$\boxed{x=3} \Rightarrow u = 3^3+2 = 29$

$$= \int_{10}^{29} \frac{\frac{1}{3}du}{u} = \frac{1}{3}\int_{10}^{29}\frac{1}{u}\,du = \frac{1}{3}\cdot\ln(u)\Big|_{10}^{29} = \frac{1}{3}\left(\ln(29) - \ln(10)\right)$$

HATÁROZOTT PARCIÁLIS INTEGRÁLÁS:

$$\int_a^b u(x)\cdot v'(x)\,dx = \Big[u(x)\cdot v(x)\Big]_a^b - \int_a^b v(x)\cdot u'(x)\,dx$$

PL: $\int_0^1 x\cdot\arctan(x)\,dx =$

$u(x)=\arctan(x)$	$v'(x)=x$
$u'(x)=\dfrac{1}{1+x^2}$	$v(x)=\dfrac{x^2}{2}$

$$=\left[\arctan(x)\cdot\frac{x^2}{2}\right]_0^1 - \int_0^1 \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx$$

$$=\frac{\arctan(1)}{2} - \frac{1}{2}\int_0^1 \frac{x^2+1-1}{x^2+1}\,dx =$$

$$=\frac{\pi/4}{2} - \frac{1}{2}\Big(\underbrace{\int_0^1 1\,dx}_{1} - \int_0^1 \frac{1}{x^2+1}\,dx\Big) =$$

$$=\frac{\pi}{8} - \frac{1}{2} + \frac{1}{2}\cdot\arctan(x)\Big|_0^1 = \frac{\pi}{8} - \frac{1}{2} + \frac{1}{2}\cdot\Big(\overbrace{\arctan(1)}^{\pi/4} - \overbrace{\arctan(0)}^{0}\Big)$$

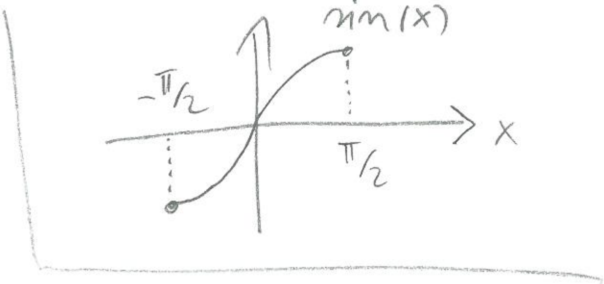
$$= \frac{\pi}{4} - \frac{1}{2}$$

151. OLDAL

PL: $\int_{-r}^{r}\sqrt{r^2-x^2}\,dx =$

$x = r\cdot\sin(u)$ $dx = r\cdot\cos(u)\,du$ $\sqrt{r^2-x^2} = r\cdot\cos(u)$	$\boxed{x=-r} \Rightarrow \sin(u)=-1 \Rightarrow u=-\pi/2$ $\boxed{x=r} \Rightarrow u=\pi/2$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} (r\cdot\cos(u))\cdot r\cdot\cos(u)\,du$$



$$= r^2\cdot\int_{-\frac{\pi}{2}}^{\pi/2}\cos^2(u)\,du =$$

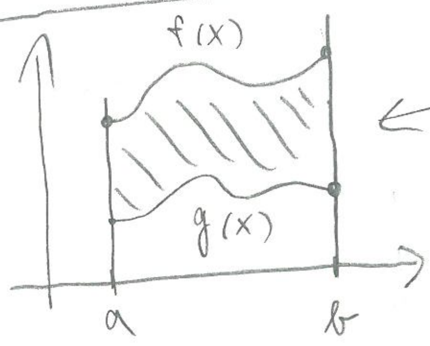
$$= r^2\cdot\int_{-\pi/2}^{\pi/2}\frac{1+\cos(2u)}{2}\,du = r^2\cdot\frac{1}{2}\cdot\pi + \underbrace{\left[\frac{1}{4}\sin(2u)\right]_{-\frac{\pi}{2}}^{\pi/2}}$$

$$\frac{1}{4}\cdot\sin(\pi) - \frac{1}{4}\sin(-\pi) = \frac{1}{4}\cdot 0 - \frac{1}{4}\cdot 0 = 0$$

$$= \frac{r^2\cdot\pi}{2} \checkmark$$

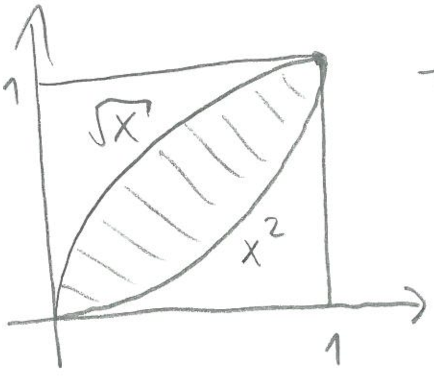
FÉLKÖR TERÜLETE (LÁSD 146. OLD)

TERÜLET-SZÁMÍTÁS



$$T = \int_a^b (f(x)-g(x))\,dx$$

PL:



$$T = \int_0^1(\sqrt{x}-x^2)\,dx = \int_0^1\sqrt{x}\,dx - \int_0^1 x^2\,dx =$$

$$= 2/3 - 1/3 = 1/3$$

LÁSD 149. OLD