$$\int \frac{1}{x \cdot \ln(x)}\,dx = \int (\ln(x))^{-1} \cdot \ln'(x)\,dx = \boxed{\begin{array}{l} u = \ln(x) \\ \frac{du}{dx} = \ln'(x) \\ du = \ln'(x)\,dx \end{array}}$$

$$= \int u^{-1}\,du = \ln|u| + C = \ln(|\ln(x)|) + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C, \text{ HISZEN}$$

$$\arctan'(x) = \frac{1}{1+x^2},$$ LÁSD 64. OLDAL.

$$\int \frac{1}{5+x^2}\,dx = \frac{1}{5}\int \frac{1}{1+\left(\frac{x}{\sqrt{5}}\right)^2}\,dx = \boxed{\begin{array}{l} u = \frac{x}{\sqrt{5}} \\ \frac{du}{dx} = \frac{1}{\sqrt{5}} \\ dx = \sqrt{5}\,du \end{array}}$$

$$= \frac{1}{5}\int \frac{1}{1+u^2}\sqrt{5}\,du = \frac{1}{\sqrt{5}}\int \frac{1}{1+u^2}\,du =$$

$$= \frac{1}{\sqrt{5}}\arctan(u) + C = \frac{1}{\sqrt{5}} \cdot \arctan\left(\frac{x}{\sqrt{5}}\right) + C$$

<u>ÁLT</u>: $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

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122. OLD.

A $\displaystyle\int \frac{1}{5+4x+x^2}\,dx = ?$ TELJES NÉGYZETTÉ ALAKÍTJUK A NEVEZŐT

$$x^2+4x+5 = x^2+2\cdot x\cdot 2+2^2+1 = (x+2)^2+1$$

$$? = \int \frac{1}{(x+2)^2+1}\,dx = \boxed{u = x+2 \quad \frac{du}{dx} = 1 \implies dx = du}$$

$$= \int \frac{1}{u^2+1}\,du = \arctan(u) + C = \arctan(x+2) + C$$

B $\displaystyle\int \frac{2x+4}{5+4x+x^2}\,dx = ?$ SZÁMLÁLÓ $= \frac{d}{dx}$ NEVEZŐ

$\boxed{u = \text{NEVEZŐ}}$ $\boxed{u = 5+4x+x^2}$ $\boxed{\frac{du}{dx} = 2x+4}$ $\boxed{(2x+4)\,dx = du}$

$$? = \int \frac{du}{u} = \ln|u| + C = \ln(5+4x+x^2) + C$$

C $$\int \frac{x-1}{5+4x+x^2}\,dx = \int \frac{\frac{1}{2}(2x+4)-3}{5+4x+x^2}\,dx =$$

$$x-1 = a\cdot(2x+4)+b \implies \boxed{a = \frac{1}{2}} \implies$$

$$x-1 = \frac{1}{2}\cdot(2x+4)+b \implies \boxed{b = -3}$$

$$= \frac{1}{2}\int \frac{2x+4}{5+4x+x^2}\,dx - 3\int \frac{1}{5+4x+x^2}\,dx =$$

$$= \frac{1}{2}\ln(5+4x+x^2) - 3\arctan(x+2) + C$$

123. OLDAL

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x) + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \operatorname{arcosh}(x) + C$$

LÁSD KÉPLETGYŰJT.

ENNEK SEGÍTSÉGÉVEL:

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = ?$$

TELJES $\square$:

$$x^2-6x = x^2-2\cdot x\cdot 3+3^2-3^2$$
$$= (x-3)^2-9$$

$$? = \int \frac{1}{\sqrt{9-(x-3)^2}}\,dx = \int \frac{1}{\sqrt{9\cdot\left(1-\left(\frac{x-3}{3}\right)^2\right)}}\,dx =$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-\left(\frac{x-3}{3}\right)^2}}\,dx =$$

$$\boxed{u = \frac{x-3}{3} \qquad \frac{du}{dx} = \frac{1}{3} \qquad dx = 3du}$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,3du = \int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C =$$

$$= \arcsin\left(\frac{x-3}{3}\right) + C$$

124. OLDAL

$$\int \frac{1}{\sqrt{x^2-6x+10}}\,dx = \int \frac{1}{\sqrt{(x-3)^2+1}}\,dx = \boxed{\begin{array}{l} u = x-3 \\ du = dx \end{array}}$$

$$= \int \frac{1}{\sqrt{u^2+1}}\,du = \operatorname{arsinh}(u) + C = \operatorname{arsinh}(x-3) + C$$

$$\int \sin^2(x)\,dx = \boxed{\text{KÉPLETGYŰJT}} = \int \frac{1-\cos(2x)}{2}\,dx =$$

$$= \int \frac{1}{2}\,dx - \frac{1}{2}\int \cos(2x)\,dx = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$$

$$\int \cos^2(x)\,dx = \int \frac{1+\cos(2x)}{2}\,dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C$$

PARCIÁLIS INTEGRÁLÁS:

$$(u\cdot v)' = u'\cdot v + u\cdot v' \Rightarrow v'\cdot u = (u\cdot v)' - u'\cdot v \Rightarrow$$

$$\Rightarrow \int v'\cdot u = \int (u\cdot v)' - \int u'\cdot v = u\cdot v - \int u'\cdot v$$

$$\boxed{\int v'(x)\cdot u(x)\,dx = u(x)\cdot v(x) - \int u'(x)\cdot v(x)\,dx}$$

125. OLDAL

$$\int v'(x)\cdot u(x)\,dx = u(x)\cdot v(x) - \int u'(x)\cdot v(x)\,dx$$

PL: $\int \underbrace{x}_{u}\cdot\underbrace{e^{-x}}_{v'}\,dx =$

$u(x)=x$	$v'(x)=e^{-x}$
$u'(x)=1$	$v(x)=-e^{-x}$

$$= \underbrace{x}_{u}\cdot\underbrace{(-e^{-x})}_{v} - \int \underbrace{1}_{u'}\cdot\underbrace{(-e^{-x})}_{v}\,dx = -x\cdot e^{-x} + \int e^{-x}\,dx =$$

$$= -x\cdot e^{-x} - e^{-x} + C = -(x+1)\cdot e^{-x} + C$$

PL: $\int \ln(x)\,dx =$

$u(x)=\ln(x)$	$v'(x)=1$
$u'(x)=\frac{1}{x}$	$v(x)=x$

$$= \overbrace{\ln(x)}^{u}\cdot\overbrace{x}^{v} - \int \overbrace{\frac{1}{x}}^{u'}\cdot\overbrace{x}^{v}\,dx = \ln(x)\cdot x - \int 1\,dx =$$

$$x\cdot(\ln(x)-1) + C$$

PL: $\int \arctan(x)\,dx =$

$u(x)=\arctan(x)$	$v'(x)=1$
$u'(x)=\frac{1}{1+x^2}$	$v(x)=x$

$$= \overbrace{\arctan(x)}^{u}\cdot\overbrace{x}^{v} - \int \overbrace{\frac{1}{1+x^2}}^{u'}\cdot\overbrace{x}^{v}\,dx = ☆$$

126.OLD

$$\star = \arctan(x)\cdot x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

SZÁMLÁLÓ = $\frac{d}{dx}$ NEVEZŐ

$$= \arctan(x)\cdot x - \frac{1}{2}\ln\left(1+x^2\right) + C$$

$$\int \cos^2(x)\cdot x^2\,dx = \int\left(\frac{1+\cos(2x)}{2}\right)\cdot x^2\,dx =$$

$$= \frac{1}{2}\int x^2\,dx + \frac{1}{2}\int \cos(2x)\cdot x^2\,dx = @$$

$$\int \underbrace{\cos(2x)}_{v'}\cdot\underbrace{x^2}_{u}\,dx =$$

$u(x) = x^2$	$v'(x) = \cos(2x)$
$u'(x) = 2x$	$v(x) = \frac{1}{2}\sin(2x)$

$$= \overbrace{x^2}^{u}\cdot\overbrace{\left(\frac{1}{2}\sin(2x)\right)}^{v} - \int \overbrace{2x}^{u'}\cdot\overbrace{\frac{1}{2}\sin(2x)}^{v}\,dx = ☺$$

$$\int \underbrace{x}_{u}\cdot\underbrace{\sin(2x)}_{v'}\,dx =$$

$u(x) = x$	$v'(x) = \sin(2x)$
$u'(x) = 1$	$v(x) = -\frac{1}{2}\cos(2x)$

$$= \overbrace{x}^{u}\cdot\overbrace{\left(-\frac{1}{2}\cos(2x)\right)}^{v} - \int \overbrace{1}^{u'}\cdot\overbrace{\left(-\frac{1}{2}\cos(2x)\right)}^{v}\,dx =$$

$$= -\frac{1}{2}x\cdot\cos(2x) + \frac{1}{4}\cdot\sin(2x) + C$$

$$☺ = \frac{1}{2}x^2\cdot\sin(2x) + \frac{1}{2}x\cdot\cos(2x) - \frac{1}{4}\cdot\sin(2x) + C$$

$$@ = \frac{1}{6}x^3 + \frac{1}{2}\cdot ☺ = \frac{1}{6}x^3 + \left(\frac{1}{4}x^2 - \frac{1}{8}\right)\cdot\sin(2x) + \frac{1}{4}\cdot x\cdot\cos(2x) + C$$

127. OLDAL

PL: $$\int \underbrace{e^x}_{v'} \cdot \underbrace{\sin(2x)}_{u}\, dx =$$

$u(x) = \sin(2x)$	$v'(x) = e^x$
$u'(x) = 2\cdot\cos(2x)$	$v(x) = e^x$

$$= \overbrace{e^x}^{v} \cdot \overbrace{\sin(2x)}^{u} - \int \overbrace{e^x}^{v} \cdot \overbrace{\left(2\cdot\cos(2x)\right)}^{u'} dx = \bigstar$$

$$\int \overbrace{e^x}^{v'} \cdot \overbrace{\cos(2x)}^{u}\, dx =$$

$u(x) = \cos(2x)$	$v'(x) = e^x$
$u'(x) = -2\cdot\sin(2x)$	$v(x) = e^x$

$$= \overbrace{e^x}^{v} \cdot \overbrace{\cos(2x)}^{u} - \int \overbrace{e^x}^{v} \cdot \left(-2\cdot\sin(2x)\right) dx$$

$$= e^x \cdot \cos(2x) + 2\cdot\int e^x \cdot \sin(2x)\, dx = ☺$$

$$\int e^x \cdot \sin(2x)\, dx = \bigstar = e^x \cdot \sin(2x) - 2\cdot ☺ =$$

$$= e^x \cdot \sin(2x) - 2\cdot e^x \cdot \cos(2x) - 4\cdot\int e^x \cdot \sin(2x)\, dx$$

ÁTRENDEZVE:

$$5\cdot\int e^x \cdot \sin(2x)\, dx = e^x \cdot \sin(2x) - 2\cdot e^x \cdot \cos(2x)$$

$$\Rightarrow \int e^x \cdot \sin(2x)\, dx = \frac{1}{5}\cdot\left(e^x \cdot \sin(2x) - 2\cdot e^x \cdot \cos(2x)\right) + C$$

128. OLDAL

POLINOMOK:

$$p(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \ldots + a_1 \cdot x + a_0$$

n-EDFOKÚ POLINOM. (KITEVŐK: NEMNEGATÍV EGÉSZ SZÁMOK)

POLINOM FAKTORIZÁCIÓJA: HA $\boxed{a_n = 1}$, AKKOR KERESÜNK OLYAN $x_1, \ldots, x_n$-ET, HOGY

$$p(x) = (x - x_1) \cdot (x - x_2) \cdot \ldots \cdot (x - x_n)$$

ALGEBRA ALAPTÉTELE: MINDEN $p(x)$-HEZ TALÁLHATÓ ILYEN $x_1, \ldots, x_n$ KOMPLEX SZÁM.

- $x_1, x_2, \ldots, x_n$: $p(x)$ GYÖKEI

PL: $p(x) = x^2 - 1 = (x+1) \cdot (x-1) = (x - x_1) \cdot (x - x_2)$

$x_1 = -1 \quad x_2 = 1$

$p(x) = x^2 + 1 = (x + i) \cdot (x - i) = x^2 - (i)^2 = x^2 - (-1)$ ✓

$x_1 = -i, \ x_2 = i$

$p(x) = x^2 - 2x + 1 = (x-1)^2$: $x_1 = x_2 = 1$: $\boxed{\text{TÖBBSZÖRÖS GYÖK}}$

$p(x) = x^2 - 3x + 2 \qquad x_{1,2} = \dfrac{3 \pm \sqrt{9 - 4 \cdot 2}}{2}$ $\nearrow x_1 = 2$, $\searrow x_2 = 1$

$p(x) = (\underset{\substack{\uparrow \\ x_1}}{x-2}) \cdot (\underset{\substack{\uparrow \\ x_2}}{x-1}) = x^2 - 2x - x + 2$ ✓

$\boxed{\text{129. OLDAL}}$