Midterm Exam - May 16, 2018, Limit thms. of probab.

Family name ——————————————— **Given name** ———————————————

Signature ——————————————— **Neptun Code** ———————————————

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (8 marks) We consider simple symmetric random walk (X_n) on $\mathbb{Z}$. Denote by T_1 the time it takes to reach level 1, i.e., $T_1 = \min\{\, n : X_n = 1\,\}$. Calculate $\mathbb{P}(T_1 = 101)$.

2. (7 marks) Let U, X and Y be independent random variables distributed as follows: $U \sim \mathrm{UNI}[0,1]$, $X, Y \sim \mathrm{EXP}(1)$. Use the method of characteristic functions to prove that

$$Z := U \cdot (X + Y) \sim \mathrm{EXP}(1).$$