7. Entropy source coding and data compression

Coding Technology

Source coding and data compression

In any text, different characters typically have different frequencies. Normal coding (without compression) means that all characters are coded using the same amount of bits.

Source coding and data compression

In any text, different characters typically have different frequencies. Normal coding (without compression) means that all characters are coded using the same amount of bits.

If we allow different characters to have varying length codewords (assigning shorter codewords to more frequent characters), it is possible to get a lower average codeword length.

Source coding and data compression

In any text, different characters typically have different frequencies. Normal coding (without compression) means that all characters are coded using the same amount of bits.

If we allow different characters to have varying length codewords (assigning shorter codewords to more frequent characters), it is possible to get a lower average codeword length.

We assume that the distribution (long-term frequency) of characters in the text is known: the probabilities of the characters are

$$p_1, \ldots, p_K,$$

where K is the size of the alphabet.

Source coding and data compression

If a coding assigns a codeword of length ℓ_k to character k, then the *average codelength* is

$$L = \sum_{k=1}^{K} p_k \ell_k.$$

Source coding and data compression

If a coding assigns a codeword of length ℓ_k to character k, then the *average codelength* is

$$L = \sum_{k=1}^{K} p_k \ell_k.$$

A coding is *prefix-free* if none of the codewords is a prefix of another codeword. This property is necessary for decoding.

Source coding and data compression

If a coding assigns a codeword of length ℓ_k to character k, then the *average codelength* is

$$L = \sum_{k=1}^{K} p_k \ell_k.$$

A coding is *prefix-free* if none of the codewords is a prefix of another codeword. This property is necessary for decoding.

The *entropy* of the text source is

$$H(X) = \sum_{k=1}^{K} p_k \log_2(1/p_k).$$

Theoretical lower bound: for any prefix-free coding,

$$L \geq H(X),$$

and the ratio $H(X)/L$ is called the *efficiency* of the code.

Shannon–Fano coding

For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil .$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Shannon–Fano coding

For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

Shannon–Fano coding

For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\ell_1 = \lceil \log_2(1/0.37) \rceil = 2, \qquad \ell_2 = \lceil \log_2(1/0.27) \rceil = 2,$$
$$\ell_3 = \lceil \log_2(1/0.24) \rceil = 3, \qquad \ell_4 = \lceil \log_2(1/0.12) \rceil = 4.$$

Shannon–Fano coding

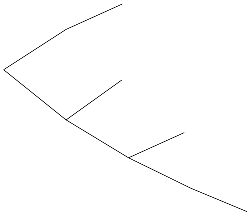
For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\ell_1 = \lceil \log_2(1/0.37) \rceil = 2, \qquad \ell_2 = \lceil \log_2(1/0.27) \rceil = 2,$$
$$\ell_3 = \lceil \log_2(1/0.24) \rceil = 3, \qquad \ell_4 = \lceil \log_2(1/0.12) \rceil = 4.$$



Shannon–Fano coding

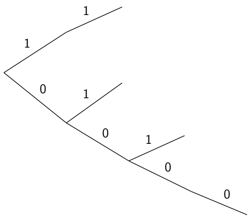
For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\ell_1 = \lceil \log_2(1/0.37) \rceil = 2, \qquad \ell_2 = \lceil \log_2(1/0.27) \rceil = 2,$$
$$\ell_3 = \lceil \log_2(1/0.24) \rceil = 3, \qquad \ell_4 = \lceil \log_2(1/0.12) \rceil = 4.$$



Shannon–Fano coding

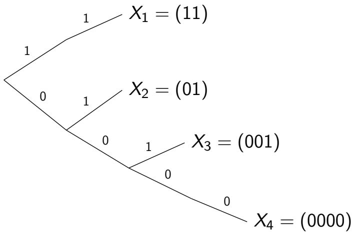
For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \ldots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\ell_1 = \lceil \log_2(1/0.37) \rceil = 2, \qquad \ell_2 = \lceil \log_2(1/0.27) \rceil = 2,$$
$$\ell_3 = \lceil \log_2(1/0.24) \rceil = 3, \qquad \ell_4 = \lceil \log_2(1/0.12) \rceil = 4.$$



Shannon–Fano coding

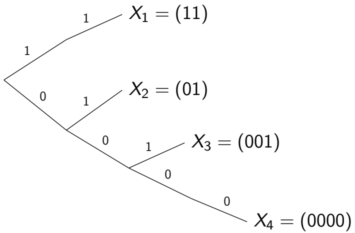
For the Shannon–Fano coding, the codeword lengths are

$$\ell_k = \lceil \log_2(1/p_k) \rceil.$$

We construct a binary tree where the depths of the leaves are $\ell_1, \dots, \ell_K$, and the codewords will be based on the route from the root to the leaves.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\ell_1 = \lceil \log_2(1/0.37) \rceil = 2, \qquad \ell_2 = \lceil \log_2(1/0.27) \rceil = 2,$$
$$\ell_3 = \lceil \log_2(1/0.24) \rceil = 3, \qquad \ell_4 = \lceil \log_2(1/0.12) \rceil = 4.$$



Symbol	Codeword
X_1	11
X_2	01
X_3	001
X_4	0000

Problem 1

Encode the following distribution using Shannon–Fano coding.

$$p_1 = 0.49, \quad p_2 = 0.14, \quad p_3 = 0.14, \quad p_4 = 0.07, \quad p_5 = 0.07,$$
$$p_6 = 0.04, \quad p_7 = 0.02, \quad p_8 = 0.02, \quad p_9 = 0.01$$

Problem 1

Encode the following distribution using Shannon–Fano coding.

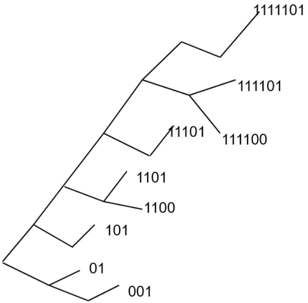
$$p_1 = 0.49, \quad p_2 = 0.14, \quad p_3 = 0.14, \quad p_4 = 0.07, \quad p_5 = 0.07,$$
$$p_6 = 0.04, \quad p_7 = 0.02, \quad p_8 = 0.02, \quad p_9 = 0.01$$

Solution. Codeword lengths: $\ell_i = \lceil \log_2 1/p_i \rceil$, so

$$\begin{aligned}
\ell_1 &= \lceil \log_2 1/p_1 \rceil = \lceil 1.029 \rceil = 2, \\
\ell_2 &= \lceil \log_2 1/p_2 \rceil = \lceil 2.836 \rceil = 3, \\
\ell_3 &= \lceil \log_2 1/p_3 \rceil = \lceil 2.836 \rceil = 3, \\
\ell_4 &= \ell_5 = 4, \quad \ell_6 = 5, \quad \ell_7 = \ell_8 = 6, \quad \ell_9 = 7.
\end{aligned}$$

(Instead of $\log_2$, the notation ld is also in use.)

Problem 1



Symbol	Codeword
X_1	01
X_2	001
X_3	101
X_4	1100
X_5	1101
X_6	11101
X_7	111100
X_8	111101
X_9	1111101

Side note: prefix-free code $\Leftrightarrow$ no codewords on inner nodes.

Problem 2

Conduct performance analysis for the previous code.

Problem 2

Conduct performance analysis for the previous code.

Solution. The entropy of the original distribution is

$$H(X) = \sum_{i=1}^{9} p_i \log_2 \left(\frac{1}{p_i} \right) = 2.314,$$

and the average codeword length for the coding is

$$\begin{aligned} L = & 0.49 \cdot 2 + 0.28 \cdot 3 + 0.28 \cdot 3 + 0.14 \cdot 4 + \\ & 0.04 \cdot 5 + 0.04 \cdot 6 + 0.01 \cdot 7 = 2.89, \end{aligned}$$

so the efficiency of the coding is

$$\frac{H(X)}{L} \approx 0.8.$$

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$p_1 = 0.37$

$p_2 = 0.27$

$p_3 = 0.24$

$p_4 = 0.12$

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\begin{array}{ll}
p_1 = 0.37 & 0.37 \\
p_2 = 0.27 & 0.27 \\
p_3 = 0.24 & 0.36 \\
p_4 = 0.12 &
\end{array}$$

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

$$\begin{array}{lll}
p_1 = 0.37 & 0.37 & 0.37 \\
p_2 = 0.27 & 0.27 & 0.63 \\
p_3 = 0.24 & 0.36 & \\
p_4 = 0.12 & &
\end{array}$$

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

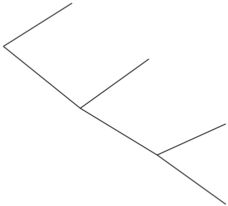
$p_1 = 0.37$	0.37	0.37	1
$p_2 = 0.27$	0.27	0.63	
$p_3 = 0.24$	0.36		
$p_4 = 0.12$			

Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

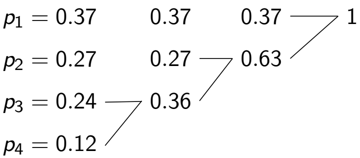
$p_1 = 0.37$	0.37	0.37	1
$p_2 = 0.27$	0.27	0.63	
$p_3 = 0.24$	0.36		
$p_4 = 0.12$			

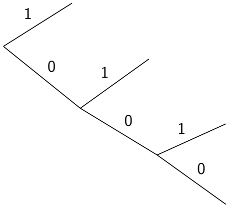


Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$

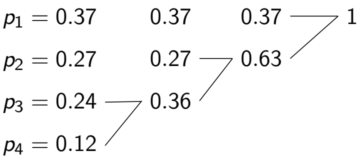


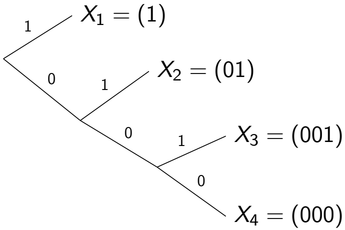


Huffman coding

Huffman coding builds the tree by adding the two smallest p_k probabilities in each step. After that, the coding works the same as for Shannon–Fano.

Example. $p_1 = 0.37, \quad p_2 = 0.27, \quad p_3 = 0.24, \quad p_4 = 0.12.$





Problem 3

Encode the source of problem 1 by Huffman coding.

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$

$p_2 = 0.14$

$p_3 = 0.14$

$p_4 = 0.07$

$p_5 = 0.07$

$p_6 = 0.04$

$p_7 = 0.02$

$p_8 = 0.02$

$p_9 = 0.01$

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49
$p_2 = 0.14$	0.14
$p_3 = 0.14$	0.14
$p_4 = 0.07$	0.07
$p_5 = 0.07$	0.07
$p_6 = 0.04$	0.04
$p_7 = 0.02$	0.02
$p_8 = 0.02$	0.03
$p_9 = 0.01$	

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49
$p_2 = 0.14$	0.14	0.14
$p_3 = 0.14$	0.14	0.14
$p_4 = 0.07$	0.07	0.07
$p_5 = 0.07$	0.07	0.07
$p_6 = 0.04$	0.04	0.04
$p_7 = 0.02$	0.02	0.05
$p_8 = 0.02$	0.03	
$p_9 = 0.01$		

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49
$p_2 = 0.14$	0.14	0.14	0.14
$p_3 = 0.14$	0.14	0.14	0.14
$p_4 = 0.07$	0.07	0.07	0.07
$p_5 = 0.07$	0.07	0.07	0.07
$p_6 = 0.04$	0.04	0.04	0.09
$p_7 = 0.02$	0.02	0.05	
$p_8 = 0.02$	0.03		
$p_9 = 0.01$			

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49	0.49
$p_2 = 0.14$	0.14	0.14	0.14	0.14
$p_3 = 0.14$	0.14	0.14	0.14	0.14
$p_4 = 0.07$	0.07	0.07	0.07	0.14
$p_5 = 0.07$	0.07	0.07	0.07	0.09
$p_6 = 0.04$	0.04	0.04	0.09	
$p_7 = 0.02$	0.02	0.05		
$p_8 = 0.02$	0.03			
$p_9 = 0.01$				

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49	0.49	0.49
$p_2 = 0.14$	0.14	0.14	0.14	0.14	0.14
$p_3 = 0.14$	0.14	0.14	0.14	0.14	0.14
$p_4 = 0.07$	0.07	0.07	0.07	0.14	0.23
$p_5 = 0.07$	0.07	0.07	0.07	0.09	
$p_6 = 0.04$	0.04	0.04	0.09		
$p_7 = 0.02$	0.02	0.05			
$p_8 = 0.02$	0.03				
$p_9 = 0.01$					

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49	0.49	0.49	0.49
$p_2 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.28
$p_3 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.23
$p_4 = 0.07$	0.07	0.07	0.07	0.14	0.23	
$p_5 = 0.07$	0.07	0.07	0.07	0.09		
$p_6 = 0.04$	0.04	0.04	0.09			
$p_7 = 0.02$	0.02	0.05				
$p_8 = 0.02$	0.03					
$p_9 = 0.01$						

Problem 3

Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49	0.49	0.49	0.49	0.49
$p_2 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.28	0.51
$p_3 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.23	
$p_4 = 0.07$	0.07	0.07	0.07	0.14	0.23		
$p_5 = 0.07$	0.07	0.07	0.07	0.09			
$p_6 = 0.04$	0.04	0.04	0.09				
$p_7 = 0.02$	0.02	0.05					
$p_8 = 0.02$	0.03						
$p_9 = 0.01$							

Problem 3

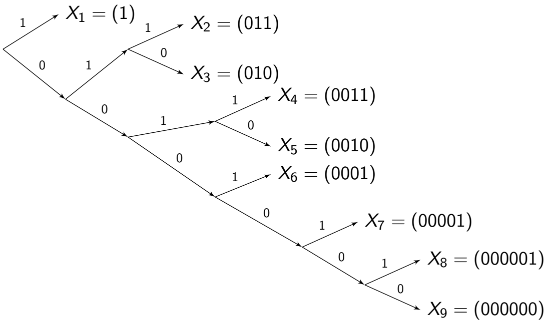
Encode the source of problem 1 by Huffman coding.

Solution. First the state graph is constructed.

$p_1 = 0.49$	0.49	0.49	0.49	0.49	0.49	0.49	0.49	1
$p_2 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.28	0.51	
$p_3 = 0.14$	0.14	0.14	0.14	0.14	0.14	0.23		
$p_4 = 0.07$	0.07	0.07	0.07	0.14	0.23			
$p_5 = 0.07$	0.07	0.07	0.07	0.09				
$p_6 = 0.04$	0.04	0.04	0.09					
$p_7 = 0.02$	0.02	0.05						
$p_8 = 0.02$	0.03							
$p_9 = 0.01$								

Problem 3

Then the code tree and coding LUT can be obtained:



Symbol	Codeword
X_1	1
X_2	011
X_3	010
X_4	0010
X_5	0011
X_6	0001
X_7	00001
X_8	000001
X_9	000000

Problem 4

Compare the performance of the Shannon-Fano coding and the Huffman coding for the previous source for sampling frequency $f_s = 160$ MHz.

Problem 4

Compare the performance of the Shannon-Fano coding and the Huffman coding for the previous source for sampling frequency $f_s = 160$ MHz.

Solution. We first compute the average codelength for both HUFF and SF coding.

$$\begin{aligned} L^{HUFF} &= 0.49 \cdot 1 + 0.14 \cdot 3 + 0.14 \cdot 3 + 0.07 \cdot 4 + 0.07 \cdot 4 + \\ &+ 0.04 \cdot 4 + 0.02 \cdot 5 + 0.02 \cdot 6 + 0.01 \cdot 6 = 2.33 \\ L^{SF} &= 0.49 \cdot 2 + 0.28 \cdot 3 + 0.14 \cdot 4 + 0.04 \cdot 5 + 0.04 \cdot 6 + \\ &+ 0.01 \cdot 7 = 2.89 \end{aligned}$$

Problem 4

Compare the performance of the Shannon-Fano coding and the Huffman coding for the previous source for sampling frequency $f_s = 160$ MHz.

Solution. We first compute the average codelength for both HUFF and SF coding.

$$\begin{aligned} L^{HUFF} &= 0.49 \cdot 1 + 0.14 \cdot 3 + 0.14 \cdot 3 + 0.07 \cdot 4 + 0.07 \cdot 4 + \\ &+ 0.04 \cdot 4 + 0.02 \cdot 5 + 0.02 \cdot 6 + 0.01 \cdot 6 = 2.33 \\ L^{SF} &= 0.49 \cdot 2 + 0.28 \cdot 3 + 0.14 \cdot 4 + 0.04 \cdot 5 + 0.04 \cdot 6 + \\ &+ 0.01 \cdot 7 = 2.89 \end{aligned}$$

At $f_s = 160$ MHz, the rates are

$$R_{HUFF} = 372.8 Mbps, \qquad R_{SF} = 462 Mbps.$$

Side note: 9 source symbols $\rightarrow$ without compression, 4 bits are required, and the rate is $R = 640 Mbps$.

Problem 5

We have a source with the following distribution and code table:

Source symbol	Probability	Codeword
X_1	0.4	0
X_2	0.2	10
X_3	0.2	110
X_4	0.2	1111

(a) Is this a prefix-free code?

(b) What is the average codelength?

(c) How far is the average codelength from the theoretical lower bound of compressibility?

(d) Is this an optimal code?

Problem 5

Solution.

(a) Yes, the code is prefix-free.

Problem 5

Solution.

(a) Yes, the code is prefix-free.

(b) $L = \sum_{i=1}^{4} p_i \ell_i = 0.4 \cdot 1 + 0.2 \cdot 2 + 0.2 \cdot 3 + 0.2 \cdot 4 = 2.2.$

Problem 5

Solution.

(a) Yes, the code is prefix-free.

(b) $L = \sum_{i=1}^{4} p_i \ell_i = 0.4 \cdot 1 + 0.2 \cdot 2 + 0.2 \cdot 3 + 0.2 \cdot 4 = 2.2.$

(c)

$$H(X) = \sum_{i=1}^{4} p_i \log_2 \left(\frac{1}{p_i} \right) = 0.4 \cdot 1.31 + 3 \cdot 0.2 \cdot 2.322 = 1.922$$

$$L - H(X) = 0.278$$

Problem 5

Solution.

(a) Yes, the code is prefix-free.

(b) $L = \sum_{i=1}^{4} p_i \ell_i = 0.4 \cdot 1 + 0.2 \cdot 2 + 0.2 \cdot 3 + 0.2 \cdot 4 = 2.2.$

(c)

$$H(X) = \sum_{i=1}^{4} p_i \log_2 \left(\frac{1}{p_i} \right) = 0.4 \cdot 1.31 + 3 \cdot 0.2 \cdot 2.322 = 1.922$$

$$L - H(X) = 0.278$$

(d) No, for X_4 the codeword 111 is sufficient instead of 1111. (The resulting code has the same codelengths as Huffman-coding, so it is optimal.)

Problem 6

Consider the source from Problem 1:

$$p_1 = 0.49, \quad p_2 = 0.14, \quad p_3 = 0.14, \quad p_4 = 0.07, \quad p_5 = 0.07,$$
$$p_6 = 0.04, \quad p_7 = 0.02, \quad p_8 = 0.02, \quad p_9 = 0.01.$$

(a) Compress the source using Shannon-Fano-Elias coding.

(b) Compute the average codelength.

(c) Compare the performance of this code with Shannon-Fano coding and Huffman coding for the same source for sampling frequency $f_s = 160$ MHz.

Problem 6

Solution.

(a)

i	p_i	$F(i)$	$\bar{F}(i)$	binary	ℓ_i	codeword
1	0.49	0	0.245	0.0011111010 ...	3	001
2	0.14	0.49	0.56	0.1000111101 ...	4	1000
3	0.14	0.63	0.7	0.1011001100 ...	4	1011
4	0.07	0.77	0.805	0.1100111000 ...	5	11001
5	0.07	0.84	0.875	0.1110000000 ...	5	11100
6	0.04	0.91	0.93	0.1110111000 ...	6	111011
7	0.02	0.95	0.96	0.1111010111 ...	7	1111010
8	0.02	0.97	0.98	0.1111101011 ...	7	1111101
9	0.01	0.99	0.995	0.1111111010 ...	8	11111110

$$F(i) = \sum_{j=0}^{i-1} p_i, \quad \bar{F}(i) = F(i) + p_i/2, \quad \ell_i = \lceil \log_2 (1/p_i) \rceil + 1$$

Problem 6

(b) Average codelength is

$$L^{SFE} = 0.49 \cdot 3 + 0.14 \cdot 4 + 0.14 \cdot 4 + 0.07 \cdot 5 + 0.07 \cdot 5 + \\ + 0.04 \cdot 6 + 0.02 \cdot 7 + 0.02 \cdot 7 + 0.01 \cdot 8 = 3.89.$$

Problem 6

(b) Average codelength is

$$L^{SFE} = 0.49 \cdot 3 + 0.14 \cdot 4 + 0.14 \cdot 4 + 0.07 \cdot 5 + 0.07 \cdot 5 + \\ + 0.04 \cdot 6 + 0.02 \cdot 7 + 0.02 \cdot 7 + 0.01 \cdot 8 = 3.89.$$

(c)

$$\begin{array}{ccc} L^{HUFF} = 2.33 & L^{SF} = 2.89 & L^{SFE} = 3.89 \\ \downarrow & \downarrow & \downarrow \\ R_{HUFF} = 372.8 Mbps & R_{SF} = 462 Mbps & R_{SFE} = 622 Mbps \end{array}$$

Recall: without coding, $R = 640 Mbps$.

Conclusion: small improvement in the average codelength L matters a lot in data speed!

Comparative analysis

performance $f_s = 160$ MHz alg. simplicity

Code	Performance	Avg. length	Data speed	Complexity
Huffman	optimal L	2.33	372.8$Mbps$	search + tree
SF	$H(X) < L < H(X) + 1$	2.89	462.4$Mbps$	tree
SFE	$H(X) + 1 < L < H(X) + 2$	3.89	622.4$Mbps$	binary conv.