(1) $c=1,\ L=3$ (1p) $f(-x)=-f(x)$ $0<x<3$; wegyd 6 szerint periodikus (3p)

$g(x)\equiv 0$ (1p)

$$u(x,t)=\frac{1}{2}\big(f(x+t)+f(x-t)\big)\Rightarrow$$

$$\Rightarrow u(0.5,2)=\frac{1}{2}\big(f(2.5)+f(-1.5)\big)\overset{(4p)}{=}\frac{1}{2}\big(f(2.5)-f(1.5)\big)= \quad (2p)$$

$$=\frac{1}{2}\big((3-2.5)-1\big)\underset{(2p)}{=}-0.25 \quad (2p)$$

---

(1) $c=1,\ L=5$ $\quad u(0.5,2)=\frac{1}{2}\big(f(2.5)-f(1.5)\big)=\frac{1}{2}(2-1.5)=0.25$

2.
$$\begin{pmatrix}1&0&2&1\\-1&2&0&1\\2&-1&3&1\\0&1&1&1\end{pmatrix}\sim\cancel{\begin{pmatrix}1&0&2&1\\0&2&2&2\\0&-1&-1&-1\\0&1&1&1\end{pmatrix}}\begin{pmatrix}1&0&2&1\\0&2&2&2\\0&-1&-1&-1\\0&1&1&1\end{pmatrix}\sim\begin{array}{c}\begin{pmatrix}1&0&2&1\\0&1&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix}\\ \uparrow\ \uparrow\qquad\quad\end{array}\quad (6p)$$

$\dim V=2$ (2p) $\Rightarrow \dim V^{\perp}=4-2=2$ (2p)

1. és 2. pivot oszlop $\Rightarrow$ $\underline{v_1}$ és $\underline{v_2}$ bázis V-ben (5p)

2B

(a)
$$\begin{pmatrix}3&-2&1&-1\\\boxed{1}&0&1&1\\0&1&1&2\\1&-1&0&1\end{pmatrix}\sim\begin{pmatrix}\boxed{1}&0&1&1\\0&-2&-2&-4\\0&\boxed{1}&1&2\\0&-1&-1&0\end{pmatrix}\sim\begin{array}{c}\downarrow\quad\downarrow\qquad\downarrow\\ \begin{pmatrix}\boxed{1}&0&1&1\\0&\boxed{1}&1&2\\\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}\\0&0&0&\boxed{2}\end{pmatrix}\end{array}\Rightarrow$$ v_1, v_2, v_4 BÁZIST ALKOTNAK

(b) $\dim V=3 \Rightarrow \dim V^{\perp}=1$

$\boxed{3A}$

$$\det\begin{pmatrix}17-\lambda & 8\\ 8 & 17-\lambda\end{pmatrix} = (17-\lambda)^2-64 = \lambda^2-34\lambda+225=0$$

$\lambda_{1,2} = \begin{cases}25\\9\end{cases}$ MINDKETTŐ POZITÍV $\overset{\text{A SZIMM.}}{\Longrightarrow}$ A POZITÍV DEFINIT 5

$\lambda_1 = 25$ $\left(\begin{array}{cc|c}-8 & 8 & 0\\ \cancel{8} & \cancel{-8} & \cancel{0}\end{array}\right)$ $v_2=t$, $v_1=t$, $\left\{\begin{pmatrix}t\\t\end{pmatrix}, t\in\mathbb{R}\right\}$, $\begin{pmatrix}\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{pmatrix}$ 5

$\lambda_2 = 9$ A SZIMMETRIKUSSÁGA MIATT $\begin{pmatrix}-\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{pmatrix}$

$$B = \begin{pmatrix}\frac{1}{\sqrt2} & -\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{pmatrix}\begin{pmatrix}5 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{pmatrix} = \begin{pmatrix}4 & 1\\ 1 & 4\end{pmatrix}$$ 5

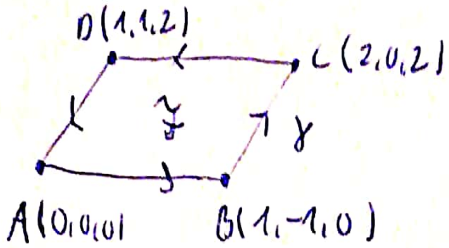
$\boxed{3B}$

$$\det\begin{pmatrix}10-\lambda & 6\\ 6 & 10-\lambda\end{pmatrix} = \lambda^2-20\lambda+64=0$$, $\lambda_{1,2} = \begin{cases}16\\4\end{cases} \Rightarrow$ A POZ. DEF.

$\lambda_1 = 16$ $\left(\begin{array}{cc|c}-6 & 6 & 0\\ \cancel{6} & \cancel{-6} & \cancel{0}\end{array}\right)$, $\left\{\begin{pmatrix}t\\t\end{pmatrix}, t\in\mathbb{R}\right\}$, $\begin{pmatrix}\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{pmatrix}$, $\lambda_2=4$, (A SZIMM.) $\begin{pmatrix}-\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{pmatrix}$

$$B = \begin{pmatrix}\frac{1}{\sqrt2} & -\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{pmatrix} = \begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}$$

4A



F : γ ÁLTAL HATÁROLT PARALELOGRAMMA FELFELÉ IRÁNYÍTVA (ÍGY LESZ γ ÉS F KOMPATIBILISEN IRÁNYÍTOTT)

$$\operatorname{curl} \vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xz-2y & z+x+\sin(y^2) & 2z+x^2+2y \end{vmatrix} = \underline{i}(2-1) - \underline{j}(2x-2x) + \underline{k}(1+2) = (1,0,3) \quad (5)$$

$$\int_\gamma \vec{F}\, d\vec{r} \overset{\text{STOKES}\ (2)}{=\!=} \iint_F \overrightarrow{\operatorname{curl}} F\, d\vec{A} \overset{\text{SPEC KÉPLET}}{=\!=} q \cdot \text{TER}(F) =$$

$\underbrace{q}_{}$ (ÁLTALÁBAN FELSZÍN)

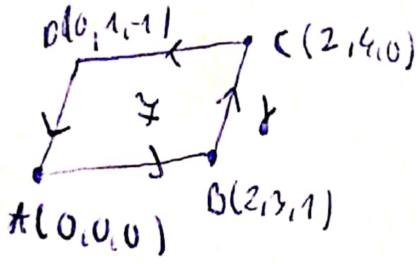
$\vec{AD} = \underline{a}$, $\vec{AB} = \underline{b}$

$\underline{a} \times \underline{b} = (2,2,-2)$

IRÁNYÍTÁS MIATT (−1)-SZERESÉT KELL VENNI

$$\frac{(1,0,3)\cdot(-2,-2,2)}{\cancel{|(-2,-2,2)|}} \cdot \cancel{|(-2,-2,2)|} = +4 \quad (8)$$

4B



F ITT IS FELFELÉ VAN IRÁNYÍTVA

$$\operatorname{curl} \vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2+3z & 2yx^2-z & e^{z^2}+x+y \end{vmatrix} = \underline{i}(1+1) - \underline{j}(1-3) + \underline{k}(4yx-4xy) = (2,2,0)$$

$$\int_\gamma \vec{F}\, d\vec{r} \overset{\text{STOKES}}{=\!=} \iint_F \overrightarrow{\operatorname{curl}} F\, d\vec{A} \overset{\text{SPEC KÉPLET}}{=\!=} q \cdot \text{TER}(F)$$

$\vec{AD} = \underline{a}$, $\vec{AB} = \underline{b}$

$\underline{a} \times \underline{b} = (4,-2,-2)$

IRÁNYÍTÁS MIATT (−1) SZERESÉT

$$\frac{(2,2,0)\cdot(-4,2,2)}{\cancel{|(-4,2,2)|}} \cdot \cancel{|(-4,2,2)|} = -4$$

5A (a) $\operatorname{curl}\vec{F}(x,y,z) \overset{(4)}{=} (0,0,0)$ + nincs szingularitás $\Rightarrow$ létezik pot fv (2)

(b) $\int -2z^3\,dx = -2z^3x + c_1(y,z)$

$\int 2ye^z + 3\,dy = y^2e^z + 3y + c_2(x,z)$

$\int y^2e^z - 6xz^2\,dz = y^2e^z - 2xz^3 + c_3(x,y)$

$f(x,y,z) = -2z^3x + y^2e^z + 3y$ jó pot. fv. (6)

$\int_\gamma \vec{F}\,d\vec{r} \overset{\text{konz. erőtér}}{=} f(C) - f(A) = \overbrace{f(1,0,0)}^{0} - \overbrace{f(1,1,0)}^{4} = -4$ (3)

5B (a) mint fent

(b) $\int e^zy - 2x\,dx = e^zyx - x^2 + c_1(y,z)$

$\int e^zx + 2yz\,dy = e^zxy + y^2z + c_2(x,z)$

$\int e^zxy + y^2\,dz = e^zxy + y^2z + c_3(x,y)$

$f(x,y,z) = e^zxy - x^2 + y^2z$ jó pot. fv.

$\int_\gamma \vec{F}\,d\vec{r} \overset{\text{konz.}}{=} f(C) - f(A) = \overbrace{f(0,1,0)}^{0} - \overbrace{f(1,-2,0)}^{-3} = 3$

6A $\operatorname{curl}\vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y - z & z^3y^2 & x+y \end{vmatrix} = \underline{i}(1 - 3z^2y^2) - \underline{j}(1+1) + \underline{k}(0 - x^2)$

$\operatorname{curl}\vec{F}(0,1,-1) = (-2,-2,0)$. Így a tanultak alapján (8)

(a) $(-2,-2,0)$-re merőlegesen választva $\underline{n}$-t nincs forgás, vagyis ha $\underline{n}$ rajta van a $-2x - 2y = 0$ síkon, pl $\underline{n} = (1,-1,0)$ ilyen (4)

(b) $\underline{n} = \operatorname{curl}\vec{F}(0,1,-1) = (-2,-2,0)$ (3)

6B

$\operatorname{curl}\vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - yz & y^2 - xz & z^2 - xy \end{vmatrix} = \underline{i}(-x + x) - \underline{j}(-y + y) + \underline{k}(-z + z) = (0,0,0)$ (8)

Bármi is az $\underline{n}$ nincs forgás (7)