4A $\underline{r}(t) = \underline{A} + t(\underline{B} - \underline{A}) = (0,2,1) + t(-1,-1,-1) = (-t, 2-t, 1-t)$ (5) $0 \le t \le 1$

$$\int_\gamma \vec{F}(\underline{r})\, d\underline{r} \overset{(6)}{=} \int_0^1 (t^2+1-t, (1-t)^2, 2-t)\cdot(-1,-1,-1)\, dt =$$

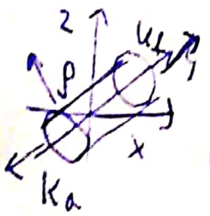
$$= \int_0^1 -t^2 - 1 + t - (1-t)^2 - 2 + t\, dt = \int_0^1 -2t^2 + 4t - 4\, dt = -\frac{2}{3} + 2 - 4 = -\frac{8}{3}$$ (4)

4B $\underline{r}(t) = (0,1,0) + t(1,-1,2) = (t, 1-t, 2t)$, $0 \le t \le 1$

$$\int_\gamma \vec{F}(\underline{r})\, d\underline{r} = \int_0^1 (t-t^2, t^2, 2t)\cdot(1,-1,2)\, dt =$$

$$= \int_0^1 t - t^2 - t^2 + 4t\, dt = \frac{1}{2} - \frac{2}{3} + 2 = \frac{11}{6}$$

5A $F = S \cup K_a$ így



2 térfogat $= 2\cdot 4\pi \cdot 2 = 16\pi$

(*) $$\iint_{K_f} \vec{g}\, d\vec{A} + \iint_F \vec{g}\, d\vec{A} \overset{GAUSS}{=} \iiint \underbrace{\operatorname{div} \vec{g}}_{2}\, dx\,dy\,dz$$

K_f befelé irányítva; hengertest (7)

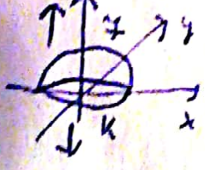
K_f paraméterezése: $\underline{r}(u,v) = (u\cdot\cos v, 1, u\cdot\sin v)$, $\boxed{0 \le u \le 2,\ 0 \le v \le 2\pi}$

$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos v & 0 & \sin v \\ -u\sin v & 0 & u\cos v \end{vmatrix} = \underline{i}(0) - \underline{j}(u) + \underline{k}(0),$$ irányítás miatt (−1)-szeresét kell venni

$$\iint_{K_f \text{ befelé}} \vec{g}\, d\vec{A} = \int_0^2\int_0^{2\pi} \overbrace{(?, 2, ?)\cdot(0,u,0)}^{2u}\, dv\,du = 8\pi$$ (7)

így (*)-ból adódik, hogy $\boxed{\iint_F \vec{g}\, d\vec{A} = 8\pi}$ (1)

5B



(**) $$\iint_{K\text{ (befelé)}} \vec{g}\, d\vec{A} + \iint_F \vec{g}\, d\vec{A} \overset{GAUSS}{=} \iiint 1\, dx\,dy\,dz = \frac{2}{3}\pi$$

félgömbtest

K paraméterezése $\underline{r}(u,v) = (u\cdot\cos v, u\cdot\sin v, 0)$, $0 \le u \le 1$, $0 \le v \le 2\pi$

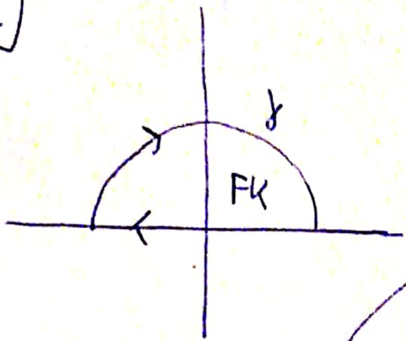
$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (0,0,u)$$ irányítás miatt (−1)-szeres kell

$$\iint_{K\text{ befelé}} \vec{g}\, d\vec{A} = \int_0^1\int_0^{2\pi} \overbrace{(?,?,-1)\cdot(0,0,-u)}^{u}\, dv\,du = +\pi,$$ így (**) miatt

$$\boxed{\iint_F \vec{g}\, d\vec{A} = -\frac{1}{3}\pi}$$

Scanned with CamScanner

6A



GREEN TÉTELBEN AZ EGYENLŐSÉG POZITÍV IRÁNYÍTÁS ESETÉN TELJESÜL ÍGY

2

$$\int_\gamma \vec{g}\, d\vec{r} \overset{GREEN}{=} -\iint_{FK} \frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial y}\, dx\,dy$$

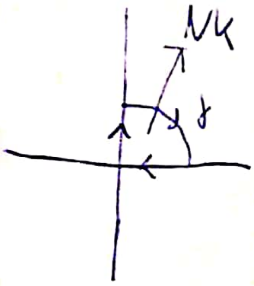
$$= -\iint_{FK} (y^2+3x^2)-(-x^2-3y^2)\,dx\,dy$$

7

$$= -\iint_{FK} 4x^2+4y^2\,dx\,dy \overset{POLÁR}{=} -\int_0^1\int_0^{\pi} 4r^2\cdot r\,d\varphi\,dr = \boxed{-\pi}$$

6

6B



$$\int_\gamma \vec{g}\, d\vec{r} \overset{GREEN}{=} -\iint_{NK} (2xy)-(4xy)\,dx\,dy$$

$$= \iint_{NK} 2xy\,dx\,dy \overset{POLÁR}{=} \int_0^1\int_0^{\frac{\pi}{2}} 2r\cos\varphi\, r\sin\varphi\cdot r\,d\varphi\,dr$$

$$= \int_0^1\int_0^{\frac{\pi}{2}} \overbrace{r^3\cdot \sin(2\varphi)}^{\text{PRIM FV } -\frac{r^3\cos(2\varphi)}{2}}\,d\varphi\,dr = \int_0^1 r^3\,dr = \boxed{\frac{1}{4}}$$