① $\alpha^2=4$, $L=3$; $f(x) = \sin \pi x + \cos\frac{\pi}{3}x \sin\frac{2\pi}{3}x = \cancel{\sin}\,\sin \pi x + \frac{1}{2}\left(\sin(\pi x) + \sin\left(\frac{\pi}{3}x\right)\right) =$ (3p)

$= \frac{1}{2}\sin\left(\frac{\pi}{3}x\right) + \frac{3}{2}\sin(\pi x)$ ~~5p~~ 4p

$f(x) = \sum_{k=1}^{\infty} A_k \sin\left(\frac{k\pi}{3}x\right) \to A_1 = 1/2,\ A_3 = 3/2$ egyébként $A_k = 0$ 4p

$u(x,t) = \sum_{k=1}^{\infty} A_k e^{-\left(\frac{k\pi}{3}\right)^2 4t} \sin\left(\frac{k\pi}{3}x\right) = \frac{1}{2} e^{-4\frac{\pi^2}{9}t} \sin\left(\frac{\pi}{3}x\right) + \frac{3}{2} e^{-4\pi^2 t} \sin(\pi x)$ 4

② $n=5$ $\quad \sum x_i = 0$ $\quad \sum x_i^2 = 16+4+0+1+25 = 46$ 3p

$\sum y_i = 4+2+1+-1+-7 = -1$ $\quad \sum x_i y_i = -16-4+0-1-35 = -56$ 3p

$\begin{pmatrix} 5 & 0 \\ 0 & 46 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ -56 \end{pmatrix}$ 2p $\to$ $a = -1/5,\ b = -\frac{56}{46} = -\frac{28}{23}$ 3p

$y = -1/5 - \frac{28}{23}x$ 3p

③ $A = \begin{pmatrix} 37 & 9 \\ 9 & 13 \end{pmatrix}$ $\quad 0 = (37-\lambda)(13-\lambda) - 81 = 481 - 50\lambda + \lambda^2 - 81 = \lambda^2 - 50\lambda + 400 =$
$= (\lambda-40)(\lambda-10)$

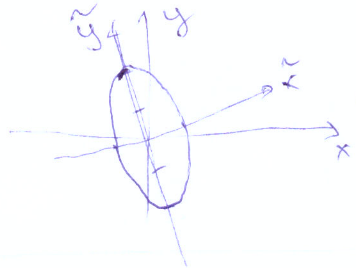
$\lambda_1 = 40;\ \lambda_2 = 10$ 4p

$\lambda_1 = 40 \to \begin{pmatrix} -3 & 9 \\ 9 & -27 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix}$ $\quad v_1 = \begin{pmatrix} t \\ t/3 \end{pmatrix} = t\begin{pmatrix} 1 \\ 1/3 \end{pmatrix}$ $\quad v_1^0 = \begin{pmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \end{pmatrix}$ 3p

$\lambda_2 = 10$ $\quad v_2^0 \perp v_1^0$ $\quad v_2^0 = \begin{pmatrix} -1/\sqrt{10} \\ 3/\sqrt{10} \end{pmatrix}$ 2p

kanonikus alak: $40\tilde{x}^2 + 10\tilde{y}^2 = 40 \iff \tilde{x}^2 + \frac{\tilde{y}^2}{4} = 1$ 4p

$a = 1 \quad b = 2$



2p

④ síkbeli $\to$ $\operatorname{curl} F = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \frac{(x^2+y^2) - 2x\cdot x}{(x^2+y^2)^2} + \frac{(x^2+y^2) - 2y y}{(x^2+y^2)^2} = 0$ 5p

(a) de! $(x,y) = (0,0)$ esetén nem értelmezett $\to$
$\to$ curl teszt inkonkluzív 4p

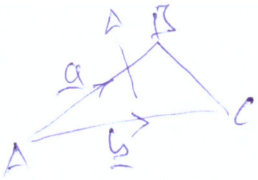
(b) $\underline{F}(\underline{r}(t)) = \left(\frac{-\sin t}{\cos^2 t + \sin^2 t}, \frac{\cos t}{\cos^2 t + \sin^2 t}\right) = (-\sin t, \cos t)$

$\dot{\underline{r}}(t) = (-\sin t, \cos t)$

$\underline{F}(\underline{r}(t)) \cdot \dot{\underline{r}}(t) = (-\sin t)^2 + (\cos t)^2 = \sin^2 t + \cos^2 t = 1$ 6p

így $\int_\gamma \underline{F}(\underline{r})\, d\underline{r} = \int_0^{2\pi} 1\, dt = 2\pi$

5.



$\underline{a} = (3,0,0) \qquad \underline{b} = (0,2,0)$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 6\underline{k}$$; felfelé mutat. 8p

$$\iint_S \underline{F}(\underline{r})\, d\underline{A} = \underline{n} \cdot \underline{F} \cdot T_S = \frac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|} \cdot \underline{F} \cdot \frac{|\underline{a} \times \underline{b}|}{2} = \frac{(\underline{a} \times \underline{b})\,\underline{F}}{2} = \frac{-18}{2} = -9$$ 7p

$(\underline{a} \times \underline{b}) = (0,0,6) \qquad \underline{F} = (1,4,-3)$

6.

Green $\Rightarrow \int_\gamma \underline{F}(\underline{r})\, d\underline{r} = \iint_D (Q'_x - P'_y)\, dx\, dy = 1 \cdot 4^2 \pi = 16\pi$ 6p

$(P,Q) = (2y, 3x) \Rightarrow Q'_x - P'_y = 3 - 2 = 1$ 5p

$\partial(D) = \gamma$, ahol D az $(1,4)$ középpontú, 4 sugarú körlap. 4p