1.) $u_{tt} = 4u_{xx} \Rightarrow c = 2$ (2p) $\quad u(x,0) = f(x) \equiv 0,\ u_t(x,0) = g(x) = \begin{cases} 3 & 0 < x \le 6 \\ 0 & \text{egyébként} \end{cases}$

$$u(x,t) = \frac{f(x+ct) + f(x-ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds = \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds \quad (3p)$$

$$u(-1,2) = \frac{1}{4}\int_{-1-4}^{-1+4} g(s)\,ds = \frac{1}{4}\int_{-5}^{3} g(s)\,ds \ (5p) = \frac{1}{4}\int_{0}^{3} 3\,ds = \boxed{9/4} \quad (5p)$$

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2) Az egyenes párhuzamos az $\underline{a} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$ vektorral. (2p)

(a) $P = \frac{1}{\underline{a}^T\underline{a}}\,\underline{a}\,\underline{a}^T$ (1p) $\qquad a^Ta = 1^2 + (-1/3)^2 = 1 + 1/9 = 10/9$ (2p)

$$aa^T = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}\begin{pmatrix} 1 & -1/3 \end{pmatrix} = \begin{pmatrix} 1 & -1/3 \\ -1/3 & 1/9 \end{pmatrix} \quad \cancel{2p}\ (3p)$$

Így: $P = \frac{1}{10/9}\begin{pmatrix} 1 & -1/3 \\ -1/3 & 1/9 \end{pmatrix} = \begin{pmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{pmatrix} = \begin{pmatrix} 0.9 & -0.3 \\ -0.3 & 0.1 \end{pmatrix}$ (3p)

(b) $\begin{pmatrix} 0.9 & -0.3 \\ -0.3 & 0.1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.9 - 0.6 \\ -0.3 + 0.2 \end{pmatrix} = \begin{pmatrix} 0.3 \\ -0.1 \end{pmatrix}$ $\cancel{5p}$ (4p)

3.) A sajátértékei: $\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0 \iff (1-\lambda)^2 - 1 = \lambda^2 - 2\lambda = 0$ (2p)

$\boxed{\lambda_1 = 0 \quad \lambda_2 = 2}$

sajátvektorok: $\lambda_1 = 0 \qquad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \to v_1 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$ (3p)

$\lambda_2 = 2 \qquad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \to v_2 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ (3p)

/vagy merőlegesség alapján/

Így: $e^A = Q e^D Q^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} e^0 & 0 \\ 0 & e^2 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} =$

$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ e^2 & e^2 \end{pmatrix} =$

$= \frac{1}{2}\begin{pmatrix} e^2+1 & e^2-1 \\ e^2-1 & e^2+1 \end{pmatrix} = \begin{pmatrix} \frac{e^2+1}{2} & \frac{e^2-1}{2} \\ \frac{e^2-1}{2} & \frac{e^2+1}{2} \end{pmatrix}$ (7p)

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4. (a) $\frac{\partial U}{\partial x} = 4x^3y^3z^2 + 2xz \Rightarrow U(x,y,z) = x^4y^3z^2 + x^2z + U_1(y,z)$

$3x^4y^2z^2 = F_2(x,y,z) = \frac{\partial U}{\partial y} = 3x^4y^2z^2 + \frac{\partial U_1}{\partial y} \Rightarrow U_1(y,z) = U_2(z)$

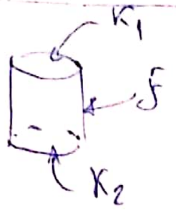
$2x^4y^3z + x^2 = F_3(x,y,z) = \frac{\partial U}{\partial z} = 2x^4y^3z + x^2 + \frac{\partial U_2}{\partial z} \Rightarrow U_2(z) = C_0$

így: $\boxed{U(x,y,z) = x^4y^3z^2 + x^2z + C_0}$ (8p) /vagy curl feltét: 7p/

(b) $\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = U(D) - U(C) = (-1)^4 2^3 2^2 + (-1)^2\cdot 2 + C_0 - \left(1^4(-1)^3 1^2 + 1^2\cdot 1 + C_0\right)$

$= 34 - 0 = \boxed{34}$ (7p) /vagy def. szerint: 8p/

5.



$S \cup K_1 \cup K_2 = S = \partial(H)$

Gauss: $\iint_S \underline{G}(\underline{r})\,d\underline{A} = \iiint_H (\operatorname{div} \underline{G})\,dx\,dy\,dz$ (2p)

$\operatorname{div} \underline{G} = \frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} + \frac{\partial G_3}{\partial z} = z + z - 2z = 0$ (3p)

így: $\iint_S \underline{G}(\underline{r})\,d\underline{A} + \iint_{K_1} \underline{G}(\underline{r})\,d\underline{A} + \iint_{K_2} \underline{G}(\underline{r})\,d\underline{A} = 0$ és: (2p)

$\cancel{\underline{G}|_{K_1} = (0,0,0)\quad \underline{G}|_{K_2} =}$ $\underline{G}|_{K_1} = (x, x, 0)$ $\quad \underline{G}|_{K_2} = (x, x, 1)$

egységnormális: $\underline{n}|_{K_1} = (0,0,1)$ $\quad \underline{n}|_{K_2} = (0,0,-1)$ (2p)

tehát: $\iint_{K_1} \underline{G}(\underline{r})\,d\underline{A} = 0$ (1p); $\quad \iint_{K_2} \underline{G}(\underline{r})\,d\underline{A} = (-1)\cdot \mathrm{Area}(K_2) = \boxed{-\pi/9}$ (3p)

$\boxed{\iint_S \underline{G}(\underline{r})\,d\underline{A}} = -\iint_{K_2} \underline{G}(\underline{r})\,d\underline{A} = \boxed{\pi/9}$ (2p)

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6. szektorszerű idom területe: $T = \int_a^b \frac{r^2(\varphi)}{2}\,d\varphi = \int_{\pi/4}^{5\pi/4} \frac{(1-\sin 2\varphi)^2}{2}\,d\varphi$ (3p)

$T = \frac{1}{2}\int_{\pi/4}^{5\pi/4} (1 - 2\sin 2\varphi + \sin^2 2\varphi)\,d\varphi = \frac{\pi}{2} - \int_{\pi/4}^{5\pi/4} \sin 2\varphi\,d\varphi + \int_{\pi/4}^{5\pi/4} \frac{\sin^2 2\varphi}{2}\,d\varphi =$ (3p)

$= \pi/2 - 0 + \int_{\pi/4}^{5\pi/4} \frac{1 - \cos 4\varphi}{4}\,d\varphi = \pi/2 + \pi/4 = \frac{3\pi}{4}$ (3p)

kiesik, vagy kimutatva felezett sin periódusra (2p)

addíciós tételből levezetve, vagy ábrából kimutatva (4p)

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