(1.) $c = 2 \quad L = 6 \quad f(x) = \begin{cases} x & 0 \le x \le 2 \\ 2 & 2 \le x \le 4 \\ 6-x & 4 \le x \le 6 \end{cases} \quad g(x) = 0$ (3p)

$u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] \Rightarrow u(1,1) = \frac{1}{2}\left[f(1+2) + f(1-2)\right] =$

(páratlan kiterjesztés) (5p) (2p) $= \frac{1}{2}\left(f(3) + f(-1)\right) = \frac{1}{2}\left(2 + -f(1)\right) = \frac{1}{2}\,1$

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(2) $B = \left\{ \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right\} \quad P = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ (2p) $P^{-1} = \frac{1}{\det P}\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$ (2p)

$[v]_B = P^{-1}[v]_T = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5+6 \\ -7-9 \end{pmatrix} = \begin{pmatrix} 11 \\ -16 \end{pmatrix}$ (6p)

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(3) $A = \begin{pmatrix} -4 & 0 \\ -3 & 5 \end{pmatrix} \quad A^T = \begin{pmatrix} -4 & -3 \\ 0 & 5 \end{pmatrix}$ (2p) $A^TA = \begin{pmatrix} -4 & -3 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -4 & 0 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 25 & -15 \\ -15 & 25 \end{pmatrix}$ (5p)

A^TA sajátértékei: $\det\begin{vmatrix} 25-\lambda & -15 \\ -15 & 25-\lambda \end{vmatrix} = 0 = (25-\lambda)^2 - (15)^2 = 625 - 50\lambda + \lambda^2 - 225 =$

$= \lambda^2 - 50\lambda + 400 = (\lambda - 10)(\lambda - 40)$ (5p)

$\lambda_1 = 10 \Rightarrow \boxed{\sigma_1 = \sqrt{10}}$; $\lambda_2 = 40 \Rightarrow \sigma_2 = \sqrt{40} = \boxed{2\sqrt{10}}$ (3p)

(4.)

$\cancel{\text{curl}\, F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \end{vmatrix}}$

(a) $\text{curl}\, F = \partial_x Q - \partial_y P = \cancel{-3\frac{\partial((x^2+y^2)y)}{\partial x} - 3\frac{\partial((x^2+y^2)x)}{\partial y}}$

$\cancel{= 3(x^2+y^2) + 9x^2} = 9x^2 + 3y^2 - (-3x^2 - 9y^2) =$ (7p)

$= 12(x^2+y^2) \neq 0$ NEM potenciális

(b) $\dot{r}(t) = (-4\sin t, 4\cos t)$

$F(r(t)) = \left(-3\left((4\cos t)^2 + (4\sin t)^2\right)4\sin t,\ 3\left((4\cos t)^2 + (4\sin t)^2\right)4\cos t\right) =$

$= (-192\sin t,\ 192\cos t)$

$\int_\gamma F(r)\,dr = \int_0^{2\pi} F(r(t)) \cdot \dot{r}(t)\,dt = \int_0^{2\pi}(768\sin^2 t + 768\cos^2 t)\,dt =$

$= 768 \cdot 2\pi = 1536\pi$ (8p)

(5) Gauss tétel $\Rightarrow$ $\iint_F \underline{G}(\underline{r})\, d\underline{A} = \iiint_K \operatorname{div} \underline{G}\, dx\,dy\,dz$ (2p)
zárt felület

$F = \partial K$

$$\operatorname{div} \underline{G} = 1 + 2z + 2z = 1 + 4z \quad (5p)$$

$$\iiint_K \operatorname{div} \underline{G}\, dx\,dy\,dz = \int_0^2 \int_0^2 \int_0^3 (1+4z)\, dx\,dy\,dz = 6 \cdot \int_0^2 (1+4z)\, dz =$$

$$= 6 \cdot \left(\left[z + 2z^2 \right]_0^2 \right) = 6 \cdot (2 + 8) = \boxed{60} \quad (8p)$$

(6.)

$$\operatorname{curl} \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial_x & \partial_y & \partial_z \\ x^2+y & z^2+xy & y^2z \end{vmatrix} = \underline{i}(2yz - 2z) - \underline{j}(0-0) + $$
$$+ \underline{k}(y-1) \quad (6p)$$

$$\operatorname{curl} \underline{F} \Big|_{(-1,2,0)} = (0,0,1) \quad (3p)$$

(a) $\underline{n} = \dfrac{\operatorname{curl} F|_P}{|\operatorname{curl} F|_P|} = (0,0,1)$ esetén maximális (2p)

(b) $\underline{n} \perp \operatorname{curl} F|_P \Leftrightarrow (\cos t, \sin t, 0)$ $\forall t \in (0, 2\pi]$
esetén nem forog (4p)