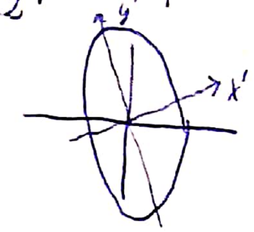
3A

$$0 = \begin{vmatrix} 17-\lambda & 6 \\ 6 & 8-\lambda \end{vmatrix} = (17-\lambda)(8-\lambda)-36 = \lambda^2 - 25\lambda + 100 \Rightarrow \lambda_{1,2} < \begin{matrix} 20 \\ 5 \end{matrix}$$ 5

$\lambda_1 = 20$ $\begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \end{pmatrix}$ 6 $\left(\begin{array}{cc|c} -3 & 6 & 0 \\ \cancel{6} & \cancel{12} & \cancel{0} \end{array}\right)$, $\lambda_1 = 5$ (SZIMM) $\begin{pmatrix} -\frac{1}{\sqrt5} \\ \frac{2}{\sqrt5} \end{pmatrix}$ $\qquad \dfrac{(x')^2}{\left(\frac12\right)^2} + \dfrac{(y')^2}{1^2} = 1$

$$20(x')^2 + 5(y')^2 = 5$$



4

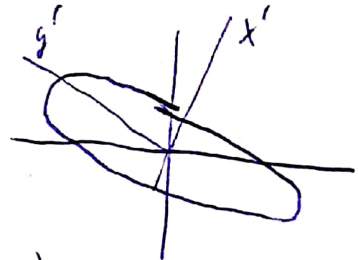
3B

$$0 = \begin{vmatrix} 13-\lambda & 16 \\ 16 & 37-\lambda \end{vmatrix} = (13-\lambda)(37-\lambda) - 16^2 = \lambda^2 - 50\lambda + 225$$

$$\lambda_{1,2} = < \begin{matrix} 45 \\ 5 \end{matrix}$$

$\lambda_1 = 45$ $\begin{pmatrix} \frac{1}{\sqrt5} \\ \frac{2}{\sqrt5} \end{pmatrix}$ $\left(\begin{array}{cc|c} -32 & 16 & 0 \\ \cancel{16} & \cancel{-8} & \cancel{0} \end{array}\right)$ $\lambda_2 = 5$ $\begin{pmatrix} -\frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \end{pmatrix}$ $\qquad \dfrac{(x')^2}{\left(\frac13\right)^2} + \dfrac{(y')^2}{1^2} = 1$

$$45(x')^2 + 5(y')^2 = 5$$



1A $\alpha = \sqrt2$, $L = 4$, $f(x) = \sin(\pi\cdot x) + \cos\left(\frac{\pi}{4}x\right)\cdot\sin\left(\frac{\pi}{2}x\right) =$ 3

$\Leftarrow \quad = \frac12 \sin\left(\frac{\pi}{4}x\right) + \frac12 \sin\left(\frac{3\pi}{4}x\right) + \sin\left(\frac{4\pi}{4}x\right)$

$A_1 = A_3 = \frac12$ 7, $A_4 = 1$, A TÖBBI EGYÜTTHATÓ 0

$$\Rightarrow \boxed{u(x,t) = \frac12 e^{-2\left(\frac{\pi}{4}\right)^2 t} \sin\left(\frac{\pi}{4}x\right) + \frac12 e^{-2\left(\frac{3\pi}{4}\right)^2 t} \sin\left(\frac{3\pi}{4}x\right) + e^{-2\pi^2 t} \cdot \sin(\pi\cdot x)}$$ 5

1B $\alpha = \sqrt5$, $L = 3$, $f(x) = \sin(\pi\cdot x) + \frac12\left(\sin(\pi x) - \sin\left(\frac{\pi}{3}x\right)\right)$

$A_1 = -\frac12$, $A_3 = 1$ $\Leftarrow \quad = -\frac12 \sin\left(\frac{\pi}{3}x\right) + \frac32 \sin(\pi\cdot x)$

$A_k = 0$ EGYÉBKÉNT $\quad u(x,t) = -\frac12 e^{-5\left(\frac{\pi}{3}\right)^2 t} \sin\left(\frac{\pi}{3}x\right) + \frac32 e^{-5\pi^2 t} \sin(\pi\cdot x)$

2A $P = \dfrac{a\cdot a^T}{|a|^2}$ $\quad a = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\;=\; \begin{pmatrix} \frac15 & -\frac25 \\ -\frac25 & \frac45 \end{pmatrix}$ 7, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ KÉPE $P\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ 8

2B $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ÍGY $P = \begin{pmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ KÉPE $P\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{7}{10} \\ \frac{21}{10} \end{pmatrix}$

4A ⓐ $\vec{F}(x,y) = (-2(x^2+y^2)y,\ 2(x^2+y^2)x)$

$$\operatorname{curl}\vec{F}(x,y) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 4x\cdot x + 2(x^2+y^2) - (-4y\,y - 2(x^2+y^2))$$ (4)

CURL $\vec{F}$ NEM AZONOSAN 0 ⟹ NINCS POTENCIÁL FV (3)

ⓑ $$\int_\gamma \vec{F}\,d\vec{r} \overset{(4)}{=} \int_0^{2\pi} (-2\cdot(9)\cdot 3\sin t,\ 2\cdot(9)\cdot 3\cdot\cos t)\cdot(-3\sin t, 3\cos t)\,dt$$

$$= \int_0^{2\pi} \overbrace{162\sin^2 t + 162\cdot\cos^2 t}^{162}\,dt = \boxed{324\pi}$$ (4)

4B ⓐ $\vec{F}(x,y) = (-4(x^2+y^2)y,\ 4(x^2+y^2)\cdot x)$

CURL $\vec{F}$ A FENTI KÉTSZERESE ⟹ NINCS POT. FV.

ⓑ $$\int_\gamma \vec{F}\,d\vec{r} = \int_0^{2\pi} (-4(25)\cdot 5\sin t,\ 4(25)\cdot 5\cdot\cos t)\cdot(-5\sin t, 5\cos t)\,dt$$ $= \boxed{5000\pi}$

6A 

$$\frac{\int_{u_p} \vec{F}\,d\vec{r}}{\text{TER}(S_p)} \approx \frac{\operatorname{curl}\vec{F}(P)\cdot \underline{n}}{|\underline{n}|}$$

↑ p KICSI

MIATT CURL $\vec{F}$ (P) IRÁNYA AZ OPTIMÁLIS (INDOKLÁS NÉLKÜL IS MAX PONT)

$$\operatorname{curl}\vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3+z^2 & x+z & x^2y \end{vmatrix} = \underline{i}(x^2-1) - \underline{j}(2xy-2z) + \underline{k}(1-3y^2)$$ (8)

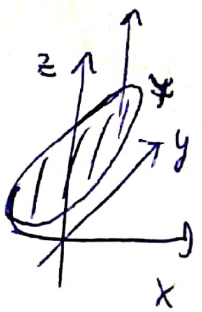
$\operatorname{curl}\vec{F}(2,-1,1) = (3, 6, -2)$ AZ OPT. IRÁNY (7)

6B $$\operatorname{curl}\vec{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z & x^3+z^2 & x+y \end{vmatrix} = i(1-2z) - \underline{j}(1-y^2) + \underline{k}(3x^2-2yz)$$

$\operatorname{curl}\vec{F}(-2,3,1) = (-1, 8, 6)$

Scanned with CamScanner

5A



$\underline{r}(u,v) = (u\cdot\cos v,\ 2u\sin v,\ 2)$, $0\le u\le 1$, $0\le v\le 2\pi$

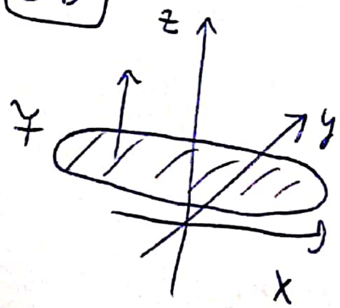
$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} i & j & k \\ \cos v & 2\sin v & 0 \\ -u\sin v & 2u\cos v & 0 \end{vmatrix} = (0,0,2u) \quad (\text{irányítás OK})$$

(6)

$$\iint_{F} \vec{F}\, d\vec{A} = \int_0^1\int_0^{2\pi} \overbrace{(?,?,4)\cdot(0,0,2u)}^{8u}\, dv\, du =$$

$$= \int_0^1 16\pi u\, du = \boxed{8\pi}$$ (9)

5B



$\underline{r}(u,v) = (3u\cdot\cos(v),\ u\cdot\sin v,\ 1)$, $0\le u\le 1$, $0\le v\le 2\pi$

$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} i & j & k \\ 3\cos v & \sin v & 0 \\ -3u\sin v & u\cdot\cos v & 0 \end{vmatrix} = (0,0,3u) \quad (\text{irányítás OK})$$

$$\iint_{F} \vec{F}\, d\vec{A} = \int_0^1\int_0^{2\pi} \overbrace{(?,?,1)\cdot(0,0,3u)}^{3u}\, dv\, du =$$

$$= \int_0^1 6\pi u\, du = \boxed{3\pi}$$