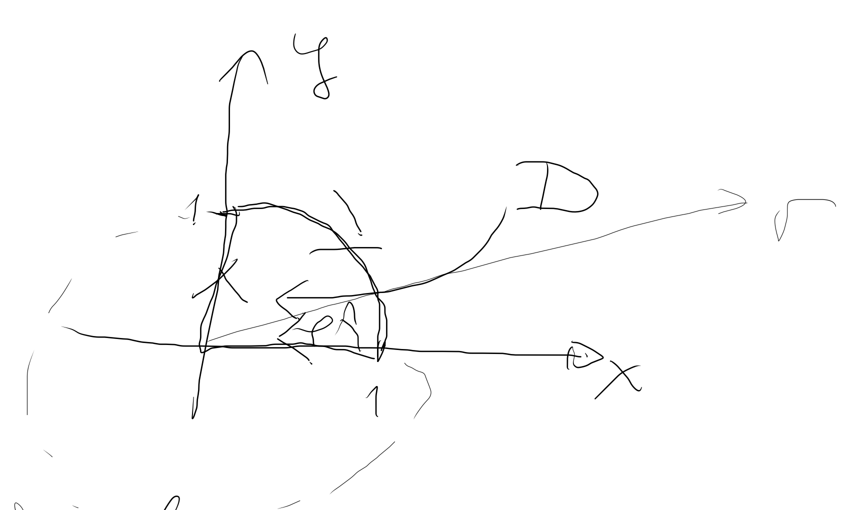
$$G(x,y) = (\underbrace{2xy^2}_{P(x,y)}, \underbrace{y^3+x^2y}_{Q(x,y)})$$



γ: az $\{x^2+y^2 \le 1,\ x \ge 0,\ y \ge 0\}$ negyedkör lemez határa, óramutató járásával egyező körüljárás

$$\int_\gamma \underline{G}(\underline{r})\, d\underline{r} = -\iint_D (Q'_x - P'_y)\, dx\, dy = -\iint (-2xy)\, dx\, dy$$

D tartomány: Green: γ a D-t határoló tartomány óramutatóval **ellentétes** irányítás,

$$\int_\gamma \underline{G}(\underline{r})\, d\underline{r} = \iint_D (Q'_x - P'_y)\, dx\, dy$$

$$Q'_x = 2xy$$

$$P'_y = 4xy$$

tehát: $$\int_\gamma \underline{G}(\underline{r})\, d\underline{r} = 2\iint_D xy\, dx\, dy = 2\int_0^1 \int_0^{\pi/2} \underbrace{r\cos\varphi \sin\varphi}_{xy}\, r\, dr\, d\varphi$$

D: $0 \le r \le 1$

$0 \le \varphi \le \pi/2$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$= \int_0^1 r^3 \int_0^{\pi/2} \sin 2\varphi \, d\varphi \, dr = \int_0^1 r^3 \left[- \frac{\cos 2\varphi}{2} \right]_0^{\pi/2} dr =$$

$\sin 2\varphi = 2 \sin\varphi \cos\varphi$

$$= \int_0^1 r^3 \left(+ \frac{1}{2} + \frac{1}{2} \right) dr = \int_0^1 \frac{r^3}{4} \, dr = \left[\frac{r^4}{16} \right]_{r=0}^{r=1} = \boxed{\frac{1}{16}}$$

$$\underline{G}(\underline{r}) = \left(x^3 + u_1(y,z) ;\ y^3 + u_2(x,z) ;\ z^3 + u_3(x,y) \right)$$

ha $\underline{G}$ sima K-n

Gauss tétel:

$$\iint_F \underline{G}(\underline{r}) \, d\underline{A} = \iiint_K (\operatorname{div} \underline{G}) \, dx\, dy\, dz$$

ha $F = \partial K$, kifelé irányítva

most: F: egységsugarú gömb] feltétele

K: — " —

$$\operatorname{div} \underline{G} = \frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} + \frac{\partial G_3}{\partial z} = 3x^2 + 3y^2 + 3z^2$$

$$\iint_F \underline{G}\, d\underline{A} = \iiint_K \underbrace{3(x^2+y^2+z^2)}_{3r^2} \underbrace{dx\,dy\,dz}_{r^2 \sin u\, dv\, du\, dr} = \int_0^1 \int_0^\pi \int_0^{2\pi} 3r^2 \sin u \cdot dv\,du\,dr \quad (*)$$

$$r^2 = x^2+y^2+z^2 = (r \sin u \cos v)^2 + (r \sin u \sin v)^2 + (r \cos u)^2 =$$

$$= r^2 \sin^2 u \sin^2 v + r^2 \sin^2 u \cos^2 v + r^2 \cos^2 u =$$

$$= r^2 \sin^2 u + r^2 \cos^2 u = \boxed{r^2}$$

$$(*) = \int_0^1 \int_0^\pi 3r^4 \sin u \underbrace{\int_0^{2\pi} dv}_{} \, du\, dr = 2\pi \int_0^1 3r^4 \int_0^\pi \sin u\, du\, dr =$$

$$= 2\pi \int_0^1 3r^4 \left[-\cos u\right]_{u=0}^{u=\pi} dr = 6\pi \int_0^1 r^4 \cdot 2\, dr = 12\pi \left[\frac{r^5}{5}\right]_{r=0}^{r=1} = \boxed{\frac{12\pi}{5}}$$

hővezetés: $u'_t = 2u''_{xx}$ $\quad 0 \le x \le 4$

$t \ge 0$

$u(x,0) = \sin(\pi x) + \cos\left(\frac{\pi}{4}x\right)\sin\left(\frac{\pi}{2}x\right)$ $\rightarrow f(x)$

$u(0,t) = u(4,t) = 0$

$\alpha^2 = 2, \quad L = 2$

cél: $f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{4}\right)$ $\quad 4 \rightarrow L$

$f(x) = \sin(\pi x) + \cos\left(\frac{\pi}{4}x\right)\sin\left(\frac{\pi}{2}x\right) = \sin \pi x + \frac{1}{2}\left(\sin\frac{3\pi}{4}x + \sin\frac{\pi}{4}x\right)$

$\frac{\pi}{4}x \to \beta$, $\quad \frac{\pi}{2}x \to \alpha$

$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right) = \frac{1}{2}\sin\left(\frac{\pi}{4}x\right) + \frac{1}{2}\sin\left(\frac{3\pi}{4}x\right) + \sin(\pi x)$

$A_1 = \frac{1}{2}; \quad A_3 = \frac{1}{2}, \quad A_4 = 1$ egyébként $A_k = 0$

így!

$$u(x,t)=\sum_{n=1}^{\infty} A_n e^{-\left(\frac{n\pi}{4}\right)^2 2t} \sin\left(\frac{n\pi x}{4}\right)=$$

$$=\frac{1}{2} e^{-\frac{\pi^2}{8}t} \sin\left(\frac{\pi}{4}x\right)+\frac{1}{2} e^{-\frac{9\pi^2}{8}t} \sin\left(\frac{3\pi}{4}x\right)+$$

$$+ e^{-2\pi^2 t} \sin(\pi x)$$

$(xy, -x^3z + e^x, z^2)$ **vektormezőt és az** $\mathcal{F}:\ \mathbf{r}(u,v) = (u\cos(v), 2u\sin(v), 2)$

$\underline{F}(\underline{r}) = (xy$

$$\underline{F}(\underline{r}) = (xy, -x^3 z + e^x, z^2)$$

$$\underline{F}(\underline{r}(u,v)) = (2u^2\cos v\sin v, -2u^3\cos^3 v + e^{u\cos v}, 2)$$

$$\underline{r}_u = (\cos v, 2\sin v, 0)$$

$$\underline{r}_v = (-u\sin v, 2u\cos v, 0)$$

$$\underline{r}_u \times \underline{r}_v = (0, 0, 2u)$$

$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos v & 2\sin v & 0 \\ -u\sin v & 2u\cos v & 0 \end{vmatrix} = \underline{k}(2u\cos^2 v + 2u\sin^2 v) = 2u\underline{k}$$

$$\underline{F}(\underline{r}(u,v))\cdot(\underline{r}_u \times \underline{r}_v) = 0 + 0 + 4u = 4u$$

$$\iint_S \underline{F}(\underline{r})\,d\underline{A} = \int_0^1 \int_0^{2\pi} 4u\,dv\,du = 2\pi \int_0^1 4u\,du = 2\pi \left[2u^2\right]_0^1 = \boxed{4\pi}$$

$$\underline{F}(x,y) = \left(-\frac{y}{x^2+y^2},\ \frac{x}{x^2+y^2}\right)$$

(a) curl test:

$$\boxed{\operatorname{curl}\underline{F} =} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{(x^2+y^2) - 2x\,x}{(x^2+y^2)^2} + \frac{(x^2+y^2) - 2y\,y}{(x^2+y^2)^2} =$$

$$= \frac{2(x^2+y^2) - 2x^2 - 2y^2}{(x^2+y^2)^2} = \boxed{0}$$

$\boxed{\text{de}}$ $(x,y) = (0,0)$ esetén nem értelmes $\underline{F}(x,y)$

a curl teszt inkonklúzív, nem tudjuk eldönteni, potenciálos-e

(b) $\underline{r}(t) = (\cos t, \sin t) \qquad 0 \le t \le 2\pi$

$$\underline{F}(\underline{r}(t)) = \left(-\frac{\sin t}{\cos^2 t + \sin t}, \frac{\cos t}{\cos^2 t + \sin^2 t} \right) = (-\sin t, \cos t)$$

$$\dot{\underline{r}}(t) = (-\sin t, \cos t)$$

$$\underline{F}(\underline{r}(t)) \cdot \dot{\underline{r}}(t) = (-\sin t)^2 + (\cos t)^2 = 1$$

$$\int_\gamma \underline{F}(\underline{r})\, d\underline{r} = \int_0^{2\pi} 1\, dt = \boxed{2\pi}$$