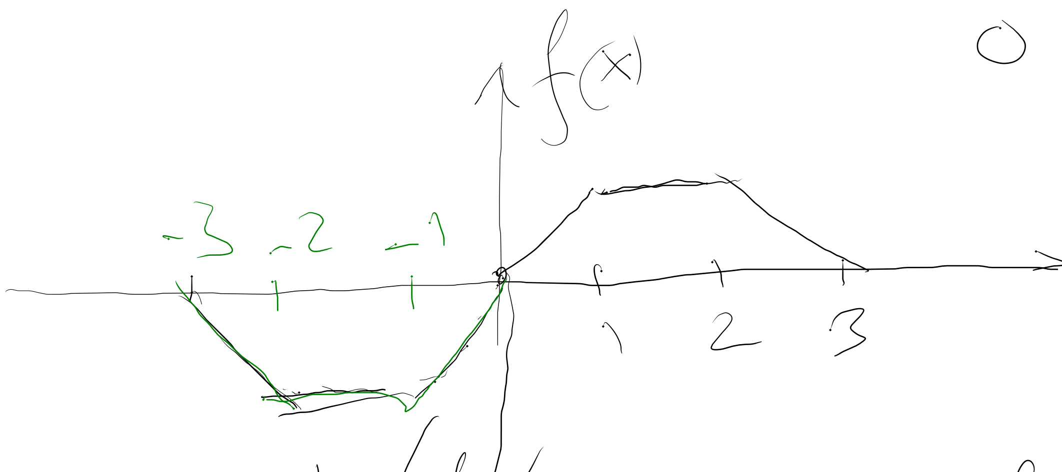
① d'Alambert formula:

$$u(x,t)=\frac{1}{2}\left[f(x+ct)+f(x-ct)\right]+\cancel{\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds}$$

most: $c=1$ $\quad g(x)\equiv 0$ $\quad f(x)$ adott $0\le x\le 3$ esetén

$L=3$

$[-L,0]$-n $f(-x)=-f(x)$



x

$$u(0.5,2)=\frac{1}{2}\left(f(0.5+2)+f(0.5-2)\right)=\frac{f(2.5)+f(-1.5)}{2}$$

t

$$=\frac{0.5-f(1.5)}{2}=\frac{0.5-1}{2}=-0.25$$

(2)

$$A=\begin{pmatrix}1&0&2&1\\-1&2&0&1\\2&-1&3&1\\0&1&1&1\end{pmatrix}\sim\begin{pmatrix}1&0&2&1\\0&2&2&2\\0&-1&-1&-1\\0&1&1&1\end{pmatrix}\sim\begin{array}{c}\begin{pmatrix}1&0&2&1\\0&1&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix}\\ \begin{matrix}\uparrow&\uparrow&&\end{matrix}\end{array}$$

1. és a 2. pivot oszlop $\Rightarrow$ $\underline{v}_1$ és $\underline{v}_2$ bázist alkot

$\dim V = 2$ $\qquad \dim V^{\perp} = 4 - \dim V = 2$

$\mathbb{R}^4$-beli vektorok

(3)

$$A=\begin{pmatrix}17&8\\8&17\end{pmatrix} \quad 0=\det(A-\lambda I)=(17-\lambda)^2-8^2=$$

$$289+\lambda^2-34\lambda-64=\lambda^2-34\lambda+225$$

$$\lambda_{1,2}=\frac{34\pm 2\sqrt{17^2-225}}{2}=17\pm 8=\begin{cases}25\\9\end{cases}$$

mivel pozitívak a sajátértékei, A poz definit

$\lambda_1 = 9 \qquad \begin{pmatrix} 17-9 & 8 \\ 8 & 17-9 \end{pmatrix} \sim \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

$v_1 = \begin{pmatrix} t \\ -t \end{pmatrix} \qquad v_1^0 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

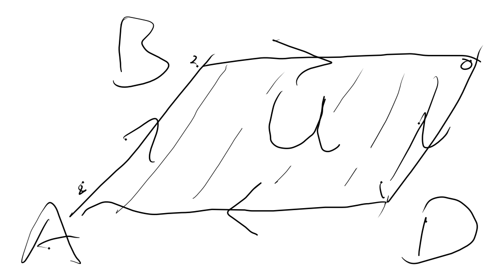
$\lambda_2 = 25 \qquad v_2^0 \perp v_1^0 \qquad v_2^0 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$Q = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \qquad Q^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

$B = Q\sqrt{D}\,Q^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} =$

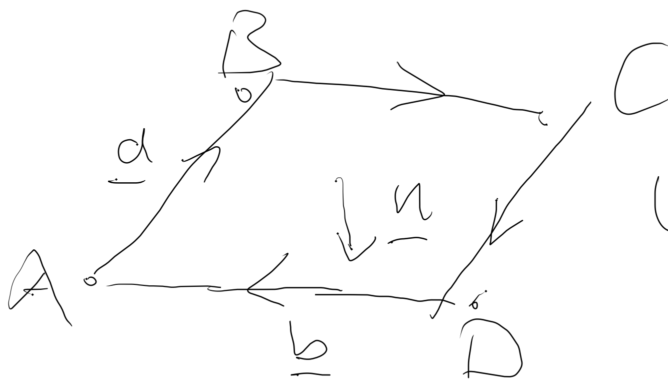
4. Stokes: $\displaystyle\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = \iint_U \text{curl}\,\underline{F} \cdot d\underline{A}$

ha $\gamma = \partial U$



$$\underline{G} = \operatorname{curl} F = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial_x & \partial_y & \partial_z \\ 2xy^2+3z & 2yx^2-z & e^{z^2}+x+y \end{vmatrix} = \underline{i}\,(1-(-1)) - \underline{j}\,(1-3) + \underline{k}\,(4xy-4xy) = (2,2,0)$$

$\boxed{\text{konstans!}}$



U síkbeli felület

$\underline{a} = \overrightarrow{AB} = (2,3,1)$

$\underline{b} = \overrightarrow{AD} = (0,1,-1)$

$\underline{a} \times \underline{b}$ vektor:

$|\underline{a} \times \underline{b}| = \text{Area}(U)$

épp az általunk megfelelő irány $\Leftarrow$ $\dfrac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|} = \underline{n}$ épp U normálvektora

$$\iint_U \overbrace{\operatorname{curl} \underline{F}}^{G}\, d\underline{A} = \underline{n} \cdot \underline{G} \cdot \text{Area}(U) = (\underline{a} \times \underline{b}) \cdot \underline{G} = \begin{vmatrix} 2 & 3 & 1 \\ 0 & 1 & -1 \\ 2 & 2 & 0 \end{vmatrix} = -6-2+2 = \boxed{-6}$$

5. $\text{curl}\,\underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial_x & \partial_y & \partial_z \\ -2z^3 & 2ye^z+3 & y^2e^z-6xz^2 \end{vmatrix} = \underline{i}\left(2ye^z - 2ye^z\right) - \underline{j}\left(-6z^2+6z^2\right) + \underline{k}\,(0-0) = \boxed{0}$

(a)

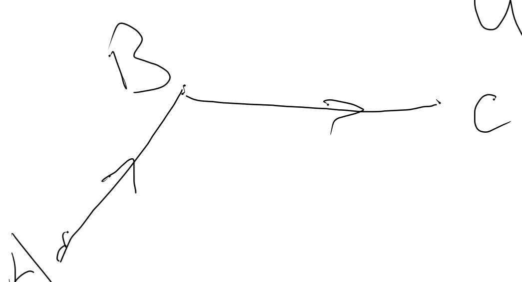
és $\underline{F}$ [illegible] $\Rightarrow \underline{F}(\underline{r})$ potenciálos

(b) $\dfrac{\partial u}{\partial x} = -2z^3 \Rightarrow u(x,y,z) = -2xz^3 + C_1(y,z)$

$\dfrac{\partial u}{\partial y} = +2ye^z+3 \Rightarrow u(x,y,z) = y^2e^z + 3y + C_2(x,z)$

$\dfrac{\partial u}{\partial z} = y^2e^z - 6xz^2 \Rightarrow u(x,y,z) = y^2e^z - 2xz^3 + C_3(x,y)$

$u(x,y,z) = y^2e^z - 2xz^3 + 3y + C$



$\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = \underset{(0,1,0)}{u(C)} - \underset{(1,-2,0)}{u(A)} = 4 + C - (4 - 6 + C) = \boxed{6}$

5.

$$\operatorname{curl} \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial x & \partial y & \partial z \\ -2z^3 & 2ye^z+3 & y^2e^z-6xz^2 \end{vmatrix} = \underline{i}\,(2ye^z - 2ye^z) - \underline{j}\,(-6z^2+6z^2) + \underline{k}\,(0-0) = \boxed{0}$$

(a)

és $\underline{F}$ mindenütt sima $\Longrightarrow$ $\underline{F}(\underline{r})$ potenciálos

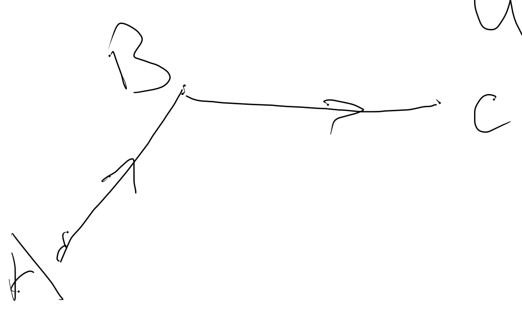
(b)

$$\frac{\partial U}{\partial x} = -2z^3 \Rightarrow U(x,y,z) = -2xz^3 + C_1(y,z)$$

$$\frac{\partial U}{\partial y} = +2ye^z+3 \Rightarrow U(x,y,z) = y^2e^z + 3y + C_2(x,z)$$

$$\frac{\partial U}{\partial z} = y^2e^z - 6xz^2 \Rightarrow U(x,y,z) = y^2e^z - 2xz^3 + C_3(x,y)$$

$$U(x,y,z) = y^2e^z - 2xz^3 + 3y + C$$



$$\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = \underset{(0,1,0)}{U(C)} - \underset{(1,-2,0)}{U(A)} = 4 + C - (4 - 6 + C) = \boxed{6}$$

⑥

$$\operatorname{curl} \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial_x & \partial_y & \partial_z \\ x^2y-z & y^2z^3 & x+y \end{vmatrix} = \underline{i}\left(1-3y^2z^2\right) - \underline{j}(1+1) + \underline{k}\left(-x^2\right)$$

$$\operatorname{curl} F\Big|_{(0,1,-1)} = \underline{i}(1-3) - \underline{j}\,2 + \underline{k}(0) = (-2,-2,0)$$

(a) a lehető leggyorsabb forgáshoz

$\underline{n} \parallel \operatorname{curl} \underline{F}$ $\qquad \underline{n} = \dfrac{(-2,-2,0)}{|(-2,-2,0)|} = \left(-\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}, 0\right)$

$|\underline{n}| = \sqrt{4+4+0} = 2\sqrt{2}$

(b) hogy ne forogjon: $\underline{n} \perp \operatorname{curl} F$

$\underline{n} = \cos\varphi\cdot(0,0,1) + \sin\varphi\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0\right)$ $\qquad$ itt $0 \le \varphi < 2\pi$ tetszőleges