$F(r) = (yx, x^2, z)$ γ: $(0,1,0)$ pontból a $(1,0,2)$-be húzott szakasz

$\int_\gamma \underline{F}(\underline{r})d\underline{r} = ?$

① paraméterezés: $r(t) = (0,1,0) + t((1,0,2) - (0,1,0)) =$

$= (0,1,0) + t(1,-1,2) =$

$= (t, 1-t, 2t)$ $0 \le t \le 1$

② görbére lokalizáljuk a vektormezőt

$\underline{F}(\underline{r}(t)) = ((1-t)t, t^2, 2t) = (t - t^2, t^2, 2t)$

③ pillanatnyi sebesség: $\dot{\underline{r}}(t) = (1, -1, 2)$

④ pillanatnyi teljesítmény:

$\underline{F}(\underline{r}(t)) \cdot \dot{\underline{r}}(t) = t - t^2 - t^2 + 4t = 5t - 2t^2$

⑤ integrálás

$$\int_\gamma \underline{F}(\underline{r})d\underline{r} = \int_0^1 \underline{F}(\underline{r}(t)) \cdot \dot{\underline{r}}(t)\,dt = \int_0^1 (5t - 2t^2)dt = \left[\frac{5t^2}{2} - \frac{2t^3}{3}\right]_{t=0}^{t=1}$$

$$= \frac{5}{2} - \frac{2}{3} = \frac{11}{6}$$

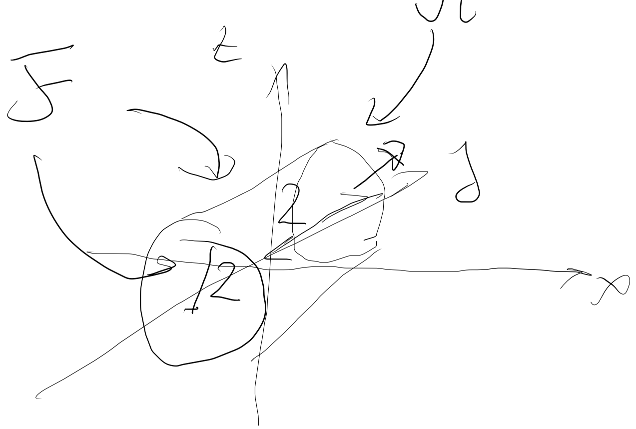
$F: \{(x,y,z) \mid x^2+z^2=4 \quad -1\le y\le 1\} \cup \{x^2+y^2\le 4,\ y=-1\}$

palást (4)

alja (A_1)

$\underline{G}(\underline{r}) = (e^y \sin z + x,\ 2,\ z - y^2 x^2)$

A: henger teteje



Gauss: $\iint_{F \cup A} \underline{G}(\underline{r})\, d\underline{A} = \iiint_H \operatorname{div} \underline{G}\, dx\,dy\,dz = 2 \cdot V_H = \boxed{16\pi}$

H: Henger

V_H: henger térfogata

$\operatorname{div} \underline{G} = 1 + 0 + 1 = 2$

$V_H = r^2\pi \cdot m = 4\pi \cdot 2 = 8\pi$

A normálvektora: $(0,1,0)$

$\underline{G}(\underline{r}) \cdot \underline{n} = 2 = \text{konst}$

$\Downarrow$

$\iint_A \underline{G}(\underline{r})\, d\underline{A} = 2 \cdot T_A = \boxed{8\pi}$

T_A: A terület

$\boxed{\iint_F \underline{G}(\underline{r})\, d\underline{A}} = 16\pi - \iint_A \underline{G}(\underline{r})\, d\underline{A} = 16\pi - 8\pi = \boxed{8\pi}$

$$\underbrace{u(x,0)}_{f(x)} = \begin{cases} x & 0 \le x \le 2 \\ 4-x & 2 \le x \le 4 \end{cases}$$

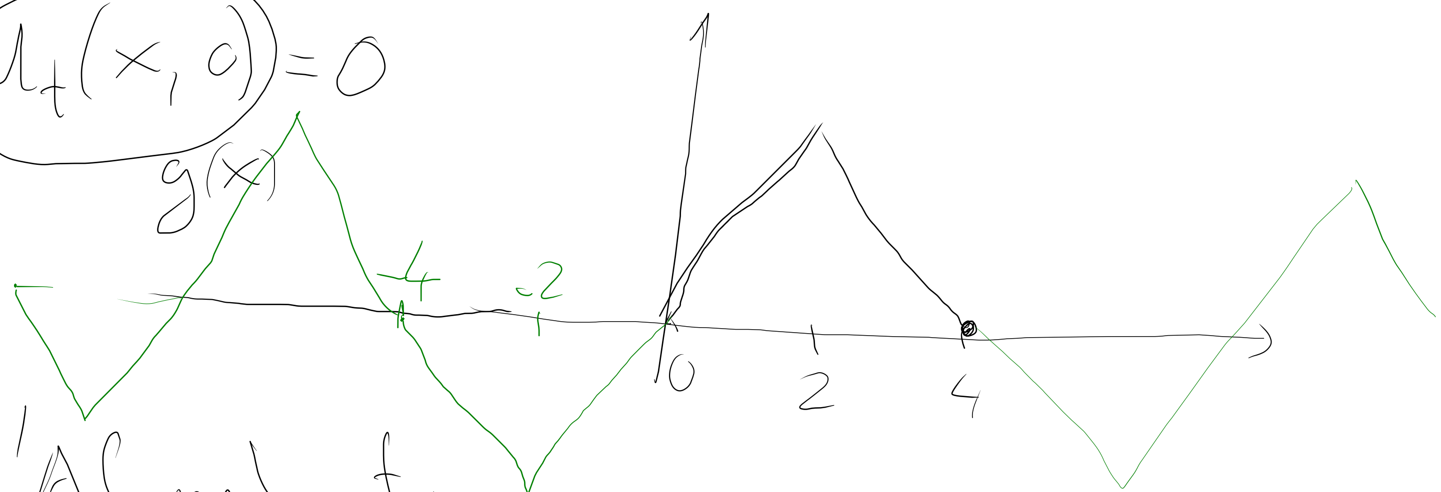
$\boxed{0 \le x \le 4}$ $\quad t > 0$

$u(x,t) = ?$

$\underbrace{u_t(x,0)}_{g(x)} = 0$

$u_{tt} = u_{xx}$

$\boxed{c = 1}$



d'Alambert:

$$u(x,t) = \frac{f(x+ct) + f(x-ct)}{2} + \cancel{\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds}$$

$$u(1,3) = \frac{f(1+1\cdot 3) + f(1-1\cdot 3)}{2} = \frac{f(4) + f(-2)}{2} = \frac{0-2}{2} = \boxed{-1}$$

$f(4) = 0$

$f(-2) = -f(2) = -2$

$\mathbb{R}^3$-ból az $x+y+z=0$ síkra vetítés mátrixa

normálvektor: $(1,1,1)$

① a sík egy bázisa: $\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}$

$$M=\begin{pmatrix}1&1\\0&-1\\-1&0\end{pmatrix} \qquad M^T=\begin{pmatrix}1&0&-1\\1&-1&0\end{pmatrix}$$

$$\begin{pmatrix}1&1\\0&-1\\-1&0\end{pmatrix}$$

$$M^TM=\begin{pmatrix}1&0&-1\\1&-1&0\end{pmatrix}\begin{pmatrix}2&1\\1&2\end{pmatrix}$$

$$(M^TM)^{-1}=\frac{1}{3}\begin{pmatrix}2&-1\\-1&2\end{pmatrix}$$

$$P=M(M^TM)^{-1}M^T=\begin{pmatrix}1&1\\0&-1\\-1&0\end{pmatrix}\frac{1}{3}\begin{pmatrix}2&-1\\-1&2\end{pmatrix}\begin{pmatrix}1&0&-1\\1&-1&0\end{pmatrix}=\frac{1}{3}\begin{pmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{pmatrix}$$

$$\begin{array}{cc} & \begin{pmatrix}1&0&-1\\1&-1&0\end{pmatrix}\\ \begin{pmatrix}2&-1\\-1&2\end{pmatrix} & \begin{pmatrix}1&1&-2\\1&-2&1\end{pmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{pmatrix}1&1&-2\\1&-2&1\end{pmatrix}\\ \begin{pmatrix}1&1\\0&-1\\-1&0\end{pmatrix} & \begin{pmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{pmatrix}\end{array}$$

$(2,0,1)$ vetülete?

$$P\begin{pmatrix}2\\0\\1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}\begin{pmatrix}2\\0\\1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3\\-3\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$A = \begin{pmatrix}9 & -2\\ -2 & 6\end{pmatrix} \qquad 0 = \det(A-\lambda I) = (9-\lambda)(6-\lambda)-4 =$$

$$= 54 - 15\lambda + \lambda^2 - 4 = \lambda^2 - 15\lambda + 50 =$$

$$= (\lambda-5)(\lambda-10)$$

$\lambda_1 = 5$

$$\begin{pmatrix}4 & -2\\ -2 & 1\end{pmatrix} \sim \begin{pmatrix}1 & -1/2\\ 0 & 0\end{pmatrix} \qquad t - 1/2\, v_y = 0$$

$$\begin{pmatrix}t\\2t\end{pmatrix} = v_1 \qquad 2t = v_y$$

$$v_1^{(0)} = \begin{pmatrix}1/\sqrt{5}\\ 2/\sqrt{5}\end{pmatrix} \qquad t^2 + 4t^2 = 1 \qquad t = \frac{1}{\sqrt{5}}$$

$\lambda_2 = 10$

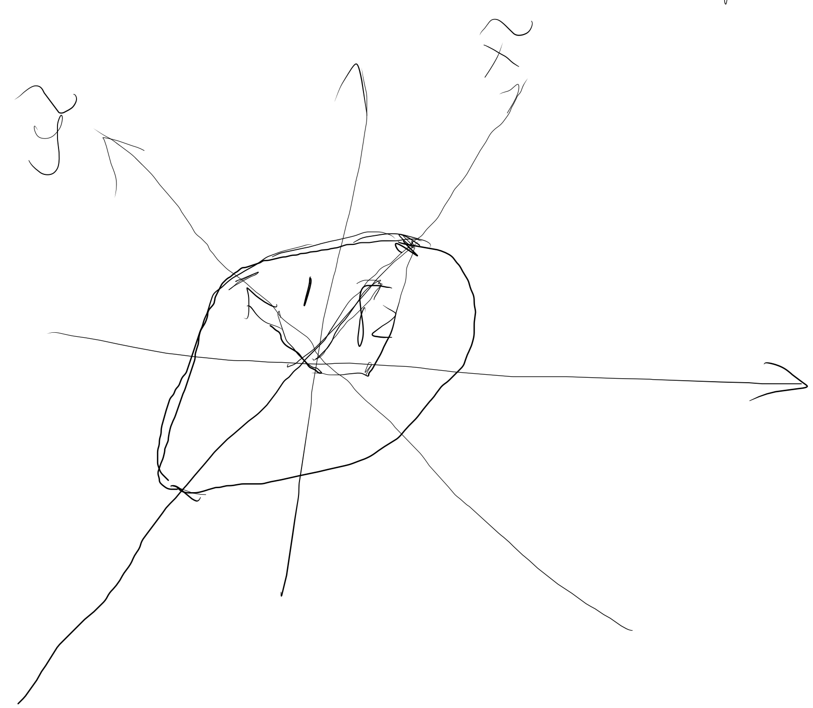
$$\begin{pmatrix}-1 & -2\\ -2 & -4\end{pmatrix} \sim \begin{pmatrix}1 & 2\\ 0 & 0\end{pmatrix} \qquad v_2 = \begin{pmatrix}t\\ -t/2\end{pmatrix} \qquad v_2^{(0)} = \begin{pmatrix}-2/\sqrt{5}\\ 1/\sqrt{5}\end{pmatrix}$$

v_1^0, v_2^0 bázisban a kvadratikus alak:

$$5\tilde{x}^2 + 10\tilde{y}^2 = 10$$

$$\frac{\tilde{x}^2}{a^2} + \frac{\tilde{y}^2}{b^2} = \frac{\tilde{x}^2}{2} + \tilde{y}^2 = 1$$ ellipszis

$$\boxed{a = \sqrt{2},\ b = 1}$$



$\underline{F}(\underline{r}) = \left(e^{x+2y+3z}, 2e^{x+2y+3z}, 3e^{x+2y+3z}\right)$

potenciales

$\operatorname{curl} \underline{F}(\underline{r}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial x & \partial y & \partial z \\ e^{x+2y+3z} & \dots & \end{vmatrix} = \dots = 0$

$\underline{F}(\underline{r})$ unirdurchol sima

$U(x,y,z) = ?$

$\frac{\partial U}{\partial x} = e^{x+2y+3z} \Rightarrow U(x,y,z) = e^{x+2y+3z} + C_1(y,z)$

$\frac{\partial U}{\partial y} = 2e^{x+2y+3z} \Rightarrow U(x,y,z) = e^{x+2y+3z} + C_2(x,z)$

$\frac{\partial U}{\partial z} = 3e^{x+2y+3z} \Rightarrow U(x,y,z) = e^{x+2y+3z} + C_3(x,y)$

$U(x,y,z) = e^{x+2y+3z} + C$

$\underline{r}_A = (0,0,0)$

$\underline{r}_B = (2\pi, 0, 0)$

$\int_\gamma \underline{F}(\underline{r})\, d\underline{r} = U(\underline{r}_B) - U(\underline{r}_A) =$

$= e^{2\pi} + C - (e^0 + C) =$

$= e^{2\pi} - 1$

$\underline{G}(\underline{r}) = (x+y,\ x-y,\ z)$

$F:\ \underline{r}(u,v) = (u,\ v,\ 2u+3v)$ $\quad 0 \le u \le 1$, $\ 0 \le v \le 1$

① lokalizáljuk $\underline{G}$-t F-re:

$\underline{G}(\underline{r}(u,v)) = (u+v,\ u-v,\ 2u+3v)$

② normálvektor: $\underline{r}_u = (1,0,2)$, $\underline{r}_v = (0,1,3)$

$$\underline{r}_u \times \underline{r}_v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{vmatrix} = -2\underline{i} - \underline{j}(3) + \underline{k} = (-2,-3,1)$$

befelé mutató irányítás:

$\underline{N} = -(\underline{r}_u \times \underline{r}_v)$-t vesszük $= (2,3,-1)$

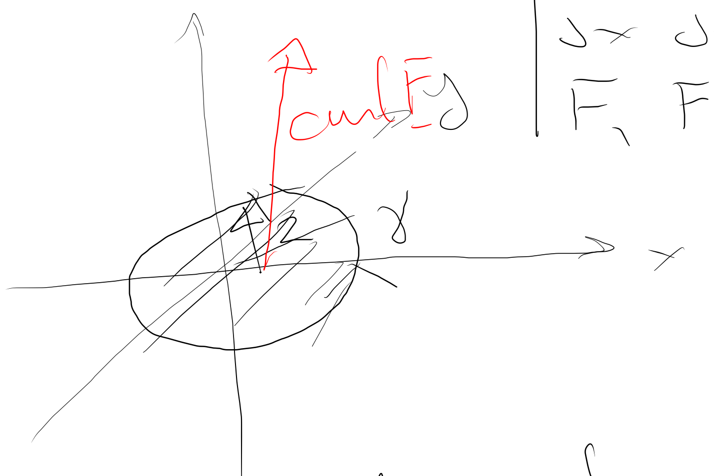
③ lokális fluxus: $\underline{G}(\underline{r}(u,v)) \cdot \underline{N}(u,v) = 2(u+v) + 3(u-v) - 1(2u+3v) = 3u + 2v$

④ $$\iint_{\mathcal{F}} \underline{G}(\underline{r})\, d\underline{A} = \int_0^1\int_0^1 \underline{G}(\underline{r}(u,v)) \cdot \underline{N}(u,v)\, du\, dv =$$

$$= \int_0^1\int_0^1 (3u+2v)\, du\, dv = \int_0^1 3u\, du + \int_0^1 2v\, dv =$$

$$= 3\left[\frac{u^2}{2}\right]_0^1 + 2\left[\frac{v^2}{2}\right]_0^1 =$$

$$= \frac{3}{2} + 1 = \boxed{\frac{5}{2}}$$



$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\underline{F}(\underline{r}) = \left(x^2y + y + e^x \sin z,\ -x + \frac{x^3}{3} - e^y,\ \sin(xyz)\right)$$

Stokes: $$\oint_\gamma \underline{F}(\underline{r})\, d\underline{r} = \iint_{\mathcal{F}} (\operatorname{curl} \underline{F})\, d\underline{A}$$

γ zárt görbe:

$\gamma = \partial\mathcal{F}$, ahol $\mathcal{F}$ a körlemez

$\underline{n} = (0,0,1)$ / koherens irányítás)

$$(\operatorname{curl} \underline{F})_z = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} =$$

$$= -1 + x^2 - (x^2 + 1) =$$

$$= \boxed{-2}$$

$u_{tt} = 9 u_{xx}$ $\boxed{c=3}$ $\boxed{L=2}$

$u(x,t) = ?$ $0 \leq x \leq 2$

$g(x) = u_t(x,0) = \sin\left(\frac{\pi}{2}x\right)\cos\left(\frac{3\pi}{2}x\right) + \sin(2\pi x)$

$\Downarrow$ $u(x,0) \equiv f(x) \equiv 0$

$$g(x) = \sum_{k=1}^{\infty} D_k \sin\left(\frac{k\pi x}{L}\right) = \sum_{k=1}^{\infty} D_k \sin\left(\frac{k\pi}{2}x\right) =$$

$$= \frac{1}{2}\left(\sin\left(\frac{\pi}{2}x + \frac{3\pi}{2}x\right) + \sin\left(\frac{\pi}{2}x - \frac{3\pi}{2}x\right)\right) + \sin 2\pi x =$$

$$= \frac{1}{2}\left[\sin(2\pi x) + \underbrace{\sin(-\pi x)}_{-\sin(\pi x)}\right] + \sin 2\pi x = \frac{3}{2}\sin 2\pi x - \frac{1}{2}\sin(\pi x)$$

$D_4 = \frac{3}{2}$ $D_2 = -1/2$ egyébként $D_k = 0$

$$u(x,t) = \sum_{r=1}^{\infty} B_r \sin\left(\frac{r\pi x}{L}\right) \sin\left(\frac{r\pi c t}{L}\right)$$

ahol $\alpha_r B_r = D_r \iff B_r = \dfrac{D_r}{\alpha_r}$

$$\alpha_r = \frac{r\pi c}{L} = \frac{3 r \pi}{2}$$

$$B_4 = \frac{D_4}{\alpha_4} = \frac{3/2}{\frac{3 \cdot 4\pi}{2}} = \frac{1}{4\pi}$$

$$B_2 = \frac{D_2}{\alpha_2} = \frac{-1/2}{\frac{3 \cdot 2\pi}{2}} = \frac{-1}{6\pi}$$

egyébként $B_r = 0$

$$u(x,t) = \frac{1}{4\pi} \sin(2\pi x) \cdot \sin(6\pi t) - \frac{1}{6\pi} \sin(\pi x) \sin(3\pi t)$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 3 & 2 \\ 2 & -2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 4 & 4 \\ 0 & -4 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

pivotoszlop.

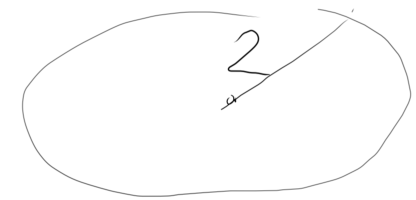
V-t kifeszíti: $\underline{v}_1$ és $\underline{v}_2$

$\dim V = 2$

$\dim V^{\perp} = 4 - \dim V = 4 - 2 = \boxed{2}$

$$\operatorname{curl}\underline{F} = (*,\ *,\ -2)$$

S



$$\underline{n} = (0,0,1)$$

$$T_S = 2^2\pi = 4\pi$$

$$(\operatorname{curl}\underline{F})\cdot\underline{n} = \boxed{-2}$$ constant

$$\iint_S \operatorname{curl}\underline{F}\cdot d\underline{A} = (-2)\,T_S = -2\cdot 4\pi = \boxed{-8\pi}$$