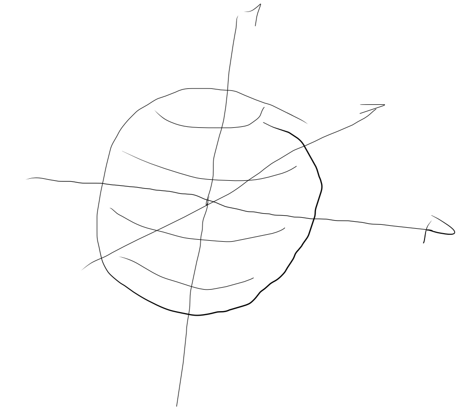
Gauss: zárt felületre $S = \partial(K)$



$$\iint_S \underline{G}(\underline{r})\, d\underline{A} = \iiint_K \mathrm{div}\,\underline{G}\; dx\,dy\,dz$$

$$\underline{G}(\underline{r}) = \left(x^3 + e^{z^2+y^2},\; y^3 + e^{xz},\; z^3 + \cos x \sin y + xy\right)$$

$$\mathrm{div}\,\underline{G} = \frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} + \frac{\partial G_3}{\partial z} = 3x^2 + 3y^2 + 3z^2 = 3r^2$$

(r az origótól vett távolság $\mathbb{R}^3$-ban)

$$\iint_S \underline{G}(\underline{r})\, d\underline{A} = \iiint_K \mathrm{div}\,\underline{G}\; dx\,dy\,dz = \int_0^1 \int_0^{\pi} \int_0^{2\pi} 3r^2 \boxed{r^2 \sin u \, dv\, du\, dr}$$

$x = r \sin u \cos v$

$y = r \sin u \sin v$

$z = r \cos u$

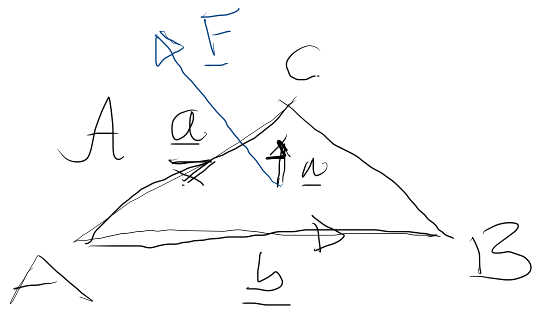
$0 \le u \le \pi$

$0 \le v \le 2\pi$

$$= 2\pi \int_0^1 \int_0^{\pi} 3r^4 \sin u \, du\, dr$$

$$= 2\pi \int_0^1 3r^4 \underbrace{\left[-\cos u\right]_{u=0}^{u=\pi}}_{-(-1)-(-1)=2} dr$$

$$= 2\pi \cdot 2 \int_0^1 3r^4\, dr = 4\pi \left[\frac{3r^5}{5}\right]_{r=0}^{r=1} = 4\pi \cdot \frac{3}{5} = \boxed{\frac{12\pi}{5}}$$



háromszögön $\underline{F}$ $\boxed{\text{konstans}}$

✓ „erőformező”

$$\iint_A \underline{F}(\underline{r})\, d\underline{A} = \underline{n} \cdot \underline{F} \cdot T(A) = \%$$

A egységnormálisa

a háromszög területe

$$\% = \frac{(\underline{a} \times \underline{b})}{\cancel{|\underline{a} \times \underline{b}|}} \cdot \underline{F} \cdot \frac{\cancel{|\underline{a} \times \underline{b}|}}{2} =$$

$$= \frac{1}{2} \underbrace{(\underline{a} \times \underline{b}) \cdot \underline{F}}_{(\underline{a},\, \underline{b},\, \underline{F})} =$$

$$\underline{a} = \overrightarrow{AB} = (5,1,7) - (3,1,7) = (2,0,0)$$

$$\underline{b} = \overrightarrow{AC} = (3,4,7) - (3,1,7) = (0,3,0)$$

$$T(A) = \frac{|\underline{a} \times \underline{b}|}{2} \qquad \underline{n} = \frac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|}$$

$$= \frac{1}{2} \begin{vmatrix} 2 & 0 & 0 \\ 6 & 3 & 0 \\ -3 & 2 & 1 \end{vmatrix} = \frac{1}{2}(2 \cdot 3 \cdot 1) = \boxed{3}$$

$L=\pi$, $f(x)$ Fourier sora (szinuszos)

$$f(x)=\sum_{k=1}^{\infty} A_k \sin(kx) = \sin 4x + \sin 6x - \sin 8x$$

$\Downarrow$

$A_4 = 1 = A_6$, $A_8 = -1$, a többi k-ra $A_k = 0$

$$u(x,t) = \sin 4x \cos 4t + \sin 6x \cos 6t - \sin 8x \cos 8t +$$

$$+ \underbrace{\qquad\qquad}_{g(x)\text{-ből jövő rész}}$$

$$\frac{\partial u}{\partial x} = 2xyz \Rightarrow u = x^2yz + u_1(y,z)$$

$$\frac{\partial u}{\partial y} = x^2z + \frac{\partial u_1}{\partial y} = x^2z + 2y$$

$$F(x,y,z) = (2xyz,\ x^2z+2y,\ x^2y-3z^2)$$

Van potenciál

$\Uparrow$

$$\text{curl test} = \quad \text{curl}\,F = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z+2y & x^2y-3z^2 \end{vmatrix} = \underline{i}\,0 + \underline{j}\,0 + \underline{k}\,0 = \underline{0}$$

$$\frac{\partial u_1}{\partial y} = 2y \Rightarrow u_1(y,z) = y^2 + u_2(z) \qquad u = x^2yz + y^2 + u_2(z)$$

$$\frac{\partial u}{\partial z} = x^2y + \frac{\partial u_2}{\partial z} = F_3(x,y,z) = x^2y - 3z^2$$

$$u_2(z) = -z^3 + C_0$$

$$\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = \int_0^1 \big(2t(1-t^2) - t^2(1+t) - 2(1-t) + t^2(1-t) - 3(1+t)^2\big)\,dt$$

$$u(x,y,z) = x^2yz + y^2 - z^3 + C_0$$

(b)

$$\int_\gamma \underline{F}(\underline{r})\,d\underline{r} = u(D) - u(C) = \underbrace{1^2\cdot 0\cdot 2 + 0^2 - 2^3 + C_0}_{u(D)} - (0^2\cdot 1\cdot 1 + 1^2 - 1^3 + C_0) =$$

$$= -8 + C_0 - (0 + C_0) = \boxed{-8}$$

$D = (1,0,2) \qquad C = (0,1,1)$

$C \to D$

VAGY

$$\underline{r}(t) = (0,1,1) + t(1,-1,1) = (t, 1-t, 1+t)$$

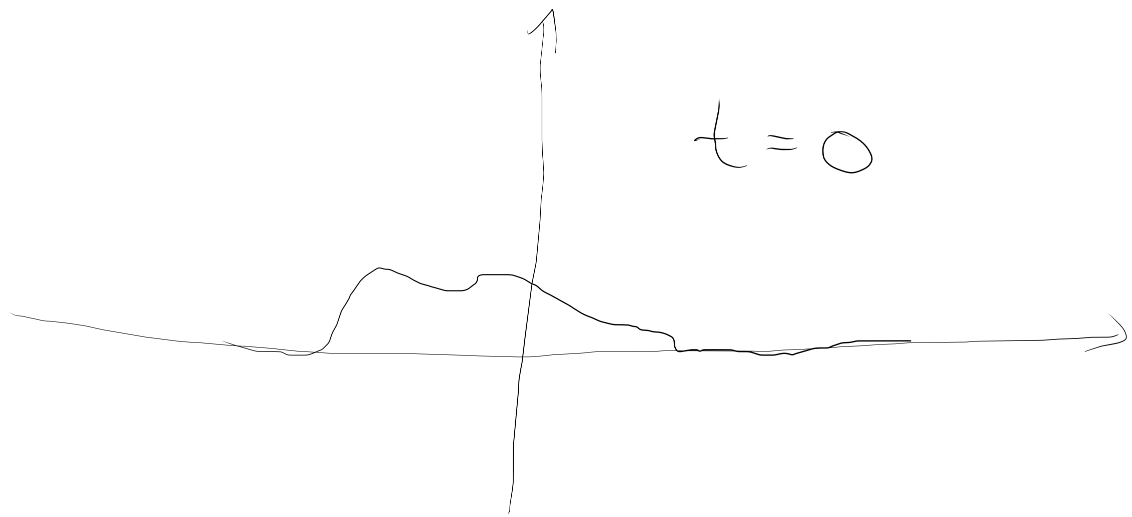
$$\overrightarrow{CD} = (1,-1,1)$$

$$\underline{F}(\underline{r}(t)) = \big(2t(1-t)(1+t),\ t^2(1+t) + 2(1-t),\ t^2(1-t) - 3(1+t)^2\big)$$

$$\underline{r}'(t) = (1,-1,1)$$

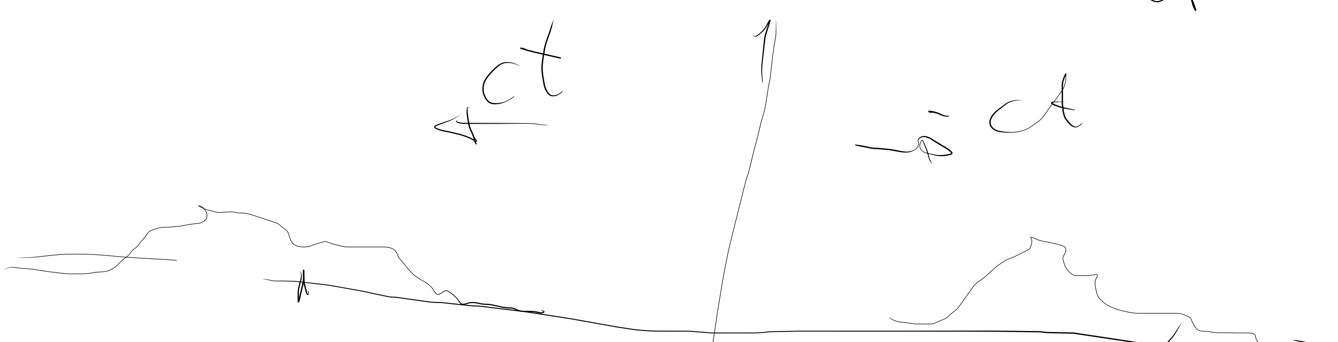
$$u(x,t) = \frac{1}{2}\left(f(x+ct) + f(x-ct) \right) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds$$

$t=0$



$$g(s) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{egyébként} \end{cases}$$

t-kor



$c=2$

$$u(0,2) = \frac{1}{4}\int_{-4}^{4} g(s)\,ds = \frac{1}{4}\int_{0}^{1} 1\,ds = \boxed{\frac{1}{4}}$$

$$\underline{F}(x,y,z) = \left(e^{x+2y+3z},\ 2e^{x+2y+3z},\ 3e^{x+2y+3z}\right)$$

$$U(x,y,z) = e^{x+2y+3z} + C_0$$

$$\underline{r}(t) = \left(t - \sin t,\ 1 - \cos t,\ t(2\pi - t)\right)$$

$t = a = 0$ esetén: $A = (0,0,0)$

$t = b = 2\pi$ -||- $B = (2\pi, 0, 0)$

$$\int_\gamma \underline{F}(\underline{r})\, d\underline{r} = U(B) - U(A) = e^{2\pi} + C_0 - (e^0 + C_0) =$$

$$= \boxed{e^{2\pi} - 1}$$